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EXPLAINING THE CONNECTION BETWEEN THE RETURNS ON FIXED-INCOME MUTUAL FUNDS AND UNDERLYING SECURITIES' YIELDS

Sanjari Chelawat

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SANJARI CHELAWAT

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ABSTRACT

EXPLAINING THE CONNECTION BETWEEN THE RETURNS ON FIXED-INCOME MUTUAL FUNDS AND UNDERLYING SECURITIES' YIELDS

Sanjari Chelawat, B.B.A. Finance

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Faculty Mentor: David Rakowski

One of the most attractive sources of income for both individual and institutional investors is fixed-income securities. However, due to the undesirable risk that arises when investing in a single security, investment in fixed-income securities is often made indirectly through fixed-income mutual funds, or funds that pool contributions of individual investors and further invest them in securities such as bonds. This study attempts to determine how returns on fixed-income securities are related to corresponding returns on fixed-income mutual funds. By using algebraic equations to ascertain bond prices and yields, the study models the return of a typical mutual fund based on the prices and yields of the underlying bonds. The study shows that bond returns are related to, but distinct from, both past and current bond yields and fund returns.

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CHAPTER 1

INTRODUCTION

The mutual fund industry of the Unites States is the largest mutual fund industry in the world per the Investment Company Fact Book (ICI, 2017). This industry serves as an attractive source of investment for individuals who are looking to invest in diversified portfolios to stabilize the growth of their retirement savings. According to the ICI, “In the year 2016, the United States mutual fund industry had total assets worth $16.3 trillion, of which bond mutual funds were the second-largest category with 22% percent of the assets” (ICI, 2017). Additionally, a significant amount of financial and human resources is spent in understanding how mutual fund markets operate and in estimating the returns received in such markets. This study analyzes how returns on a fixed-income mutual fund compare to the prices and returns on its underlying fixed-income securities.

Recording the relationship between returns on mutual funds holding fixed-income securities such as bonds and returns on the underlying bonds themselves will be important to individual investors. Extension of the existing research will allow financial analysts to precisely explain this relationship, and understanding it will allow investors to better judge the trade-offs involved between investing in bonds or the funds that hold them. While investing directly in bonds allows investors to potentially earn a higher return by avoiding fund management fees, it also exposes investors to considerable risks through reduced diversification. Investing in mutual funds, on the other hand, provides investors with lesser returns but allows them to take the inherent advantage of diversification that mutual funds provide, which makes for lesser risk.

Understanding how bond returns affect the returns of their holding funds will also allow academia to better conduct research on mutual funds. Moreover, numerous websites around the internet provide investors with advice regarding investment in either bonds or their holding funds. By attempting to understand the relationship between these two variables, this study seeks to determine if advice provided in this area is correct or flawed. This will allow investors and their investment advisors to better forecast when and how the returns that investors earn on a fixed-income mutual fund actually approximate the more-easily observed yields on the bonds held by the fund.

This study is organized as follows: Chapter 2 provides background information on mutual funds, bonds, and the scholarship relevant to the study. Chapter 3 describes the methodology used in the study along with the models developed. Chapter 4 provides an example of the applications developed in the models, while Chapter 5 makes conclusions of the analysis. Appendix A presents a detailed explanation of the computations involved in calculating bond returns.

CHAPTER 2

LITERATURE REVIEW

2.1 Mutual Funds

Mutual funds are financial instruments that pool money from individual investors and invest it in a portfolio of securities with the aim of generating income and/or capital gains. Fidelity Investments defines mutual funds as “investment strategies that allow individuals to pool money together with other investors to purchase a collection of stocks, bonds, or other securities that might be difficult to recreate on their own” (Fidelity Investments, 2017). According to Investopedia, “A mutual fund's portfolio is structured and maintained to match the investment objectives stated in its prospectus” (Investopedia, 2017).

Mutual funds can be priced and traded with several different mechanisms. Open-end mutual funds are priced at the end of each trading day according to their net asset value (NAV). NAV is the sum of weighted average of underlying funds’ asset value. At the end of each trading day, open-end funds issue new shares to investors who wish to “buy” shares and then redeem shares from investors who wish to “sell.” The U.S. Securities and Exchange Commission’s (SEC) gives guidance on the exchange process for mutual funds. According to the SEC, “Mutual funds are required by law to price their shares each business day and they typically do so after the major U.S. exchanges close. This price—the per-share value of the mutual fund’s assets minus its liabilities—is called the NAV or net asset value. Mutual funds must sell and redeem their shares at the NAV that is calculated after the investor places a purchase or redemption order. This means that, when an investor places a purchase order for mutual fund shares during the day, the investor won't know what the purchase price is until the next NAV is calculated" (U.S. SEC, 2017).

Mutual funds can invest in a diverse variety of securities. The type of security held is one way of classifying mutual funds. However, they do face restrictions on some strategies, such as short-selling, investing in derivatives, and taking on leverage. Mutual funds investing in either only equity or only debt instruments are called equity funds or fixed-income funds, respectively. Balanced funds invest in both equity and debt instruments. Money Market Mutual Funds (MMMFs) invest in highly liquid, short-term securities, such as treasury bills and commercial paper.

2.2 Bonds

The SEC states that "bonds are considered debt securities; borrowers issue bonds to raise money from investors willing to lend them money for a certain amount of time" (U.S. SEC, 2017). According to the *Wall Street Journal*, bonds are essentially "loans, or IOUs, but the investor serves as the bank. The investor loans money to a company, a city, the government – and the borrower promises to pay the lender back in full, possibly with regular interest payments" (*Wall Street Journal*, 2017). Bonds are mostly issued with a maturity period ranging from one to ten years. A bond indenture refers to a legal contract between bond issuers and bond investors which contains information regarding the face value, maturity date, coupon rate, seniority and type of the bond. Another concept related to bonds is the yield, which refers to the discount rate that makes the present value of all expected future cash flows equal to the bond's current price. The bond's price is determined by the market forces of demand and supply in the financial markets. Additionally, a bond's yield and its price are inversely related. According to the FINRA website, the return on a bond can be calculated using the following formula:

Figure 2.1: Formula for Total Return

$$\textbf{Total Return} = \frac{\left(\left(\text{End Value of Principal} + \text{Coupon Interest} + \text{Compound Interest}\right) - (\text{Taxes} + \text{Fees/Commission}) - \text{Beginning Value of Principal}\right)}{\text{Beginning Value of Principal}} \times 100$$

The goal of this research is to evaluate the relationship between returns on mutual funds and the returns on underlying securities held by those funds. The research focuses on mutual funds holding fixed-income securities such as bonds.

2.3 Scholarship Relevant to the Study

Many researchers have attempted to understand the returns received on mutual funds by analyzing the returns received on the underlying securities held by the fund. Work has been performed for either equity funds, fixed-income funds, or balanced funds (those that invest in both debt and equity securities). The existing literature has provided evidence of why investors behave in a certain manner, to what extend the risk-return trade-offs explain their investing behaviors, and some indications of the relationship that exists between mutual fund returns and returns on underlying securities.

Cornell et al. (1991) extend the literature on performance of low-grade bond funds and analyze the “pricing of low-grade bonds by examining the performance of low-grade bond funds” (Cornell & Green, 1991). The authors use a two-factor model to adjust for risk, and conclude that “the returns on low-grade bond funds are not statistically different from the returns on high-grade bonds” (Cornell & Green, 1991). The authors summarize that “the findings reveal that over the long run low-grade bond fund returns are approximately equal to the returns provided by an index of high-grade bonds" (Cornell & Green, 1991). However, they provide no formal and direct comparison(s) of the returns earned on individual bonds and the corresponding returns on the fund or index that is composed of those bonds.

Dale L. Domian at the University of Saskatchewan in Canada, along with William Reichenstein at Baylor University in Texas, examined taxable bond fund returns to understand the efficiency of U.S. taxable bond markets. They concluded that the predictions of the efficient market hypothesis explained the bond fund returns. They concluded that "consistent negative relationship exists between expense ratios and one-year through five-year net returns, and that, on average, a 1% higher expense ratio reduces returns by about 1%, although the reduction may be slightly smaller for high-grade, short-term funds" (Domain and Reichenstein, 2002). They suggested that in a diversified investment portfolio, investors must try and choose a low-cost bond fund to increase his/her investment returns.

Carhart (1997) takes a different approach to demonstrate that "persistence in mutual fund performance does not reflect superior stock-picking skill. Rather, common factors in stock returns and persistent differences in mutual fund expenses and transaction costs explain almost all the predictability in mutual fund returns" (Carhart, 1997). Carhart concluded that "expenses have at least a one-for-one negative impact on fund performance, and that turnover also negatively impacts performance. Secondly, fund performance and load fees are strongly and negatively related, probably due to higher total transaction costs for load funds" (Carhart, 1997). He also concluded that higher one-year returns on mutual funds cannot be explained by the momentum strategies applied by fund managers, but "because some mutual funds just happen by chance to hold relatively larger positions in last year's winning stocks" (Carhart, 1997). Through his study, he expands "the existing literature by controlling for survivor bias, and by documenting common-factor and cost-based explanations for mutual fund persistence" (Carhart, 1997).

This research focuses on analyzing the relation between bond mutual fund returns and the returns on the underlying bonds held by the mutual fund. Blake et al. (1993) support the use of debt instruments to understand the investment performance of mutual funds by stating that "there are fewer influences affecting bond funds, and therefore the likelihood of measuring and understanding their performance is greater" (Blake, Elton & Gruber, 1993). Under their research, a sample of two bonds were examined using linear and nonlinear models to eliminate survivorship bias, and to show that "modeling is much simpler for bond mutual funds than for stock mutual funds and that bond fund performance is robust across a wide range of models" (Blake, Elton & Gruber, 1993). Blake et al. also note that minimal literature exists on the performance of bond funds even though they constitute a major part of the mutual fund industry. This research aims to understand a similar relationship between bond returns and bond funds' returns, but is different as it involves creating a simple fund and comparing the bond fund returns to average bond yields and returns. Additionally, the research mainly focuses on the debt side rather than the equity side while analyzing the mutual funds' performance, and does not emphasize on the equity stock returns, expenses, and transactions costs. Thus, considering the existing literature and adding new aspects to the mutual fund performance analysis will allow investors to take better investment decisions and will make it easier for them to find suitable securities among the large volume of securities present in the financial markets.

CHAPTER 3

THE MODELS

Mutual funds act as indirect investment vehicles that invest in underlying securities to earn returns. Thus, it seems plausible that there exists a relation between returns received on mutual funds and those received on underlying securities. The following models relate mutual fund returns to underlying bonds' prices and yields.

3.1 Methodology

A mutual fund's return over a period is calculated as the percentage change in its Net Asset Value (NAV), which is equal to the net value gained from all assets that the fund holds in their respective proportions. The value of an asset, for the purposes of this paper, can be defined as the worth of the asset at a point of time. Since bond investments typically include provisions for periodic payments, the value of a bond is equal to the cumulative cash flows received to date plus the price at which the bond can be sold at that date. If the bond is held until maturity, the final price equals the face value, which is a lump sum paid at maturity.

The study employs algebraic methods to relate bond prices and yields to mutual fund returns. Model I depicts a mutual fund's returns in terms of underlying bond prices. Model II establishes an empirically developed assumption between bond returns and mutual fund returns, and then expresses bond returns in terms of yield.

3.2 Model I

Table 3.1 depicts relevant notations for Model I along with their respective explanations. The mutual fund's investment in underlying bonds occurs at the start of period *1*. Since the formulae before depict end-of-period values, the bonds are assumed to be purchased at the end of period *0*, which refers to the period before the mutual fund's investment in underlying bonds.

Table 3.1: Relevant Notations for Model I

$Bond_{b,t}$	Bond b at the end of period t
$P_{b,t}$	Price of bond b at the end of period t
$PMT_{b,t}$	Coupon payment received on bond b at the end of period t
$v_{b,t}$	Value of investment in bond b at the end of period t
w_b	Weight of bond b in the mutual fund
$NAV_{fund,t}$	Net Asset Value of fund at the end of period t
$r_{fund,t}$	Cumulative return on fund over the period t-1 to t

The value of the fund's investment in underlying bonds at the end of period *0* is equal to the respective beginning prices of the bonds, since no coupon payments have yet been received:

$$v_{1,0} = P_{1,0} \,, \tag{1}$$

and

$$v_{2,0} = P_{2,0} \,. \tag{2}$$

The NAV of the fund at the end of period *0* is equal to the sum of the respective weighted averages of the values of the underlying bonds:

$$NAV_{fund,0} = w_1 v_{1,0} + w_2 v_{2,0} \,. \tag{3}$$

By substituting the values of $v_{1,0}$ and $v_{2,0}$ from formulae (1) and (2) respectively in formula (3), the NAV of the fund at the end of period *0* can be given as:

$$NAV_{fund,0} = w_1 P_{1,0} + w_2 P_{2,0} \ . \tag{4}$$

The value of the fund's investment in underlying bonds at the end of period *1* is equal to the respective prices of the bonds at the end of period *1* plus the coupon payment for period *1*:

$$v_{1,1} = PMT_{1,1} + P_{1,1} \ , \tag{5}$$

and

$$v_{2,1} = PMT_{2,1} + P_{2,1} \ . \tag{6}$$

The NAV of the fund at the end of period *1* is equal to the sum of the respective weighted averages of the values of the underlying bonds:

$$NAV_{fund,1} = w_1 v_{1,1} + w_2 v_{2,1} \ . \tag{7}$$

By substituting the values of $v_{1,1}$ and $v_{2,1}$ from formulae (5) and (6) respectively in formula (7), the NAV of the fund at the end of period *1* can be given as below:

$$NAV_{fund,1} = w_1 (PMT_{1,1} + P_{1,1}) + w_2 (PMT_{2,1} + P_{2,1}) \ . \tag{8}$$

The cumulative return on the mutual fund over period *1* is given by the percentage change in the fund's NAV over period *1*, and is calculated as follows:

$$r_{fund,1} = \frac{NAV_{fund,1} - NAV_{fund,0}}{NAV_{fund,0}} \ . \tag{9}$$

The value of the fund's investment in underlying bonds at the end of period *2* is equal to the respective prices of the bonds at the end of period *2* plus the coupon payment for period *1* and period *2*:

$$v_{1,2} = PMT_{1,1} + PMT_{1,2} + P_{1,2} \ , \tag{10}$$

and

$$v_{2,2} = PMT_{2,1} + PMT_{2,2} + P_{2,2} \ . \tag{11}$$

The NAV of the fund at the end of period *2* is equal to the sum of the respective weighted averages of the values of the underlying bonds:

$$NAV_{fund,2} = w_1 v_{1,2} + w_2 v_{2,2} \ . \quad (12)$$

By substituting the values of $v_{1,2}$ and $v_{2,2}$ from formulae (10) and (11) respectively in formula (12), the NAV of the fund at the end of period *2* can be given as below:

$$NAV_{fund,2} = w_1(PMT_{1,1} + PMT_{1,2} + P_{1,2}) + w_2(PMT_{2,1} + PMT_{2,2} + P_{2,2}) \ . \quad (13)$$

The cumulative return on the mutual fund over period *2* is given by the percentage change in the fund's NAV over period *2*, and is calculated as follows:

$$r_{fund,2} = \frac{NAV_{fund,2} - NAV_{fund,0}}{NAV_{fund,0}} \ . \quad (14)$$

By substituting the values of $NAV_{fund,2}$, $NAV_{fund,1}$, and $NAV_{fund,0}$ from formulae (13), (8), and (4) respectively in formulae (9) and (14), the cumulative returns on the mutual fund over period *1* and *2* respectively can be given as below:

$$r_{fund,1} = \frac{w_1(PMT_{1,1}+P_{1,1}-P_{1,0})+w_2(PMT_{2,1}+P_{2,1}-P_{2,0})}{w_1P_{1,0}+w_2P_{2,0}} \ , \quad (15)$$

and $$r_{fund,2} = \frac{w_1(PMT_{1,1}+PMT_{1,2}+P_{1,2}-P_{1,0})+w_2(PMT_{2,1}+PMT_{2,2}+P_{2,2}-P_{2,0})}{w_1P_{1,0}+w_2P_{2,0}} \ . \quad (16)$$

Formulae (15) and (16) summarize Model I, where the fund's return over time is expressed in terms of the prices and payments received from underlying bonds.

3.3 Model II

Table 3.2 depicts relevant notations for Model II along with their respective explanations. The return on mutual funds is explained by the change in its net asset value. It is plausible that the returns on a mutual fund can be explained by the weighted average of the underlying bonds' returns. Although no concrete demonstration for this relationship exists to our knowledge, empirical testing of this relationship for the purposes of this study revealed that the weighted average of the underlying bonds' returns was close to the return on the mutual fund. Model II has been developed based on taking this empirically tested relationship as an assumption. As stated before, the bonds are assumed to be purchased at the end of period *0*.

The computations used to develop formulae (17) - (29) are detailed in Appendix A.

Table 3.2: Relevant Notations for Model II

$Bond_{b,t}$	Bond *b* at the end of period *t*
$P_{b,t}$	Price of bond *b* at the end of period *t*
$PMT_{b,t}$	Coupon payment received on bond *b* at the end of period *t*
$y_{b,t}$	Yield to Maturity on bond *b* for period *t*
CR_b	Coupon rate on bond *b*
$r_{b.t}$	Cumulative return on fund at the end of period *t*
w_b	Weight of bond *b* in the mutual fund

The price of bond *1* at end of period *0* is computed as the sum of discounted coupon payments received in period *1* and period *2* and the face value of bond *1* at the yield-to-maturity for period *1*:

$$P_{1,0} = \frac{PMT_{1,1}}{(1+y_{1,1})} + \frac{PMT_{1,2}}{(1+y_{1,1})^2} + \frac{FV_{1,2}}{(1+y_{1,1})^2}\,. \tag{17}$$

The price of bond *1* at the end of period *1* is computed as the sum of discounted coupon payments received in period *2* and the face value of bond *1* at the yield-to-maturity for period *2*:

$$P_{1,1} = \frac{PMT_{1,2}}{(1+y_{1,2})^1} + \frac{FV_{1,2}}{(1+y_{1,2})^1}\,. \tag{18}$$

The return received on bond *1* over period *1* is computed as the sum of coupon payment received in period *1* and the change in bond price over period *1,* divided by the beginning bond price:

$$r_{1,1} = \frac{PMT_{1,1} + P_{1,1} - P_{1,0}}{P_{1,0}} \,. \tag{19}$$

By substituting values of $P_{1,0}$ and $P_{1,1}$ from formulae (17) and (18) respectively in formula (19), the return on bond *1* over period *1* can be given as follows.

For a zero-coupon bond, the formula for return on bond *1* over period *1* is:

$$r_{1,1} = \frac{(1+y_{1,1})^2}{(1+y_{1,2})} - 1 \,. \tag{20}$$

For a bond with coupon payments, the formula for return on bond *1* over period *1* is:

$$r_{1,1} = \frac{1+\frac{1+\frac{1}{CR_1}}{1+y_{1,2}}}{\frac{1}{1+y_{1,1}}+\frac{1+\frac{1}{CR_1}}{(1+y_{1,1})^2}} - 1 \,. \tag{21}$$

The price of bond *1* at end of period *2* should equal the face value of bond *1*:

$$P_{1,2} = FV_{1,2} \,. \tag{22}$$

The return received on bond *1* over period *2* is computed as the sum of cumulative coupon payments received on bond *1* and the change in bond price over period *2*, divided by the beginning bond price:

$$r_{1,2} = \frac{PMT_{1,1} + PMT_{1,2} + FV_{1,2} - P_{1,0}}{P_{1,0}} \,. \tag{23}$$

By substituting the value of $P_{1,0}$ from formula (17) in formula (23), the return on bond *1* over period *2* can be given as follows.

For a zero-coupon bond, formula for return on bond *1* over period *2*:

$$r_{1,2} = (1 + y_{1,1})^2 - 1 \,. \tag{24}$$

For a bond with coupon payments, formula for return on bond *1* over period *2*:

$$r_{1,2} = \left[\frac{2+\frac{1}{CR_1}}{2+y_{1,1}+\frac{1}{CR_1}} * \left(1+y_{1,1}\right)^2\right] - 1\ . \tag{25}$$

Using the same expressions and by changing the notations for bond *b* from *1* to *2*, the formulae for calculating return on bond *2* can be given as follows.

For a zero-coupon bond, formula for return on bond *2* over period *1*:

$$r_{2,1} = \frac{(1+y_{2,1})^2}{(1+y_{2,2})} - 1\ . \tag{26}$$

For a bond with coupon payments, formula for return on bond *2* over period *1*:

$$r_{2,1} = \frac{1+\frac{1+\frac{1}{CR_2}}{1+y_{2,2}}}{\frac{1}{1+y_{2,1}}+\frac{1+\frac{1}{CR_2}}{(1+y_{2,1})^2}} - 1\ . \tag{27}$$

For a zero-coupon bond, formula for return on bond *2* over period *2*:

$$r_{2,2} = (1+y_{2,1})^2 - 1\ . \tag{28}$$

For a bond with coupon payments, formula for return on bond *2* over period *2*:

$$r_{2,2} = \left[\frac{2+\frac{1}{CR_2}}{2+y_{2,1}+\frac{1}{CR_2}} * \left(1+y_{2,1}\right)^2\right] - 1\ . \tag{29}$$

Finally, the cumulative returns on the mutual fund over period *1* and *2* respectively can be given as below:

$$r_{fund,1} \approx (w_1 * r_{1,1}) + (w_2 * r_{2,1})\ , \tag{30}$$

and

$$r_{fund,2} \approx (w_1 * r_{1,2}) + (w_2 * r_{2,2})\ . \tag{31}$$

As evident, the returns on a mutual fund for different periods can be related to the underlying bonds' respective yields-to-maturity, under the assumption that the returns on the fund can be, at least in part, explained by the sum of the weighted averages of the returns of underlying bonds. However, it is important to note that while the return on fund is related to the yields or returns on the underlying bonds, the fund’s returns will generally not be equal to but approximate the returns on the underlying bonds.

CHAPTER 4

AN ILLUSTRATION

The following illustration depicts an application of Model I and Model II. A hypothetical mutual fund holding one bond each of two bonds, with a maturity period of two years and face value of $1,000 was developed. Bond *1* is a zero-coupon bond, whereas bond *2* has a coupon rate of 7.00% payable annually. It is assumed that the respective weightage of the fund's investment in both bonds remains constant over all periods.

Table 4.1 and Table 4.2 list the relevant data used for both bonds over two periods. It is widely known that bond prices are determined by the market forces of demand and supply. The yields are then calculated based on bond prices. Accordingly, bond prices are assumed in the illustration, and relevant yields-to-maturity are calculated based on assumed bond prices for the two bonds.

Table 4.1: Relevant Data for Bond *1*

Bond *1*	Period *0*	Period *1*	Period *2*
Starting price		$800.00	$920.00
Coupon rate (CR)	0.00%	0.00%	0.00%
TTM	2	2	1
PPY	1	1	1
FV	$1,000.00	$1,000.00	$1,000.00
PV		$800.00	$920.00
PMT		$0.00	$0.00
Nper		2	1
FV		$1,000.00	$1,000.00
rate		11.803%	8.696%
Yield (y)		11.803%	8.696%
Ending market price	$800.00	$920.00	$1,000.00

Table 4.2: Relevant Data for Bond *2*

Bond *2*	Period *0*	Period *1*	Period *2*
Starting price		$950.00	$960.00
Coupon rate (CR)	7.00%	7.00%	7.00%
TTM	2	2	1
PPY	1	1	1
FV	$1,000.00	$1,000.00	$1,000.00
PV		$950.00	$960.00
PMT		$70.00	$70.00
Nper		2	1
FV		$1,000.00	$1,000.00
rate		9.876%	11.458%
Yield (y)		9.876%	11.458%
Ending market price	$950.00	$960.00	$1,000.00

4.1 Model I

Table 4.3 states the NAV of the mutual fund for each period by applying formulae (4), (8), and (13), and the cumulative returns on the mutual fund by applying formulae (15) and (16) illustrated in Model I in chapter 3.

Table 4.3: NAV and Cumulative Return Values Calculated Using Model I

Mutual Fund	Period *0*	Period *1*	Period *2*
Weight Bond *1* (w_1)	50%	50%	50%
Weight Bond *2* (w_2)	50%	50%	50%
NAV	\$875.00	\$975.00	\$1,070.00
Cumulative Return ($r_{fund,t}$)		11.429%	22.286%

The NAV values and Cumulative Return values are calculated in the following manner using the formulae from Model I and data from Table 4.1 and 4.2:

$$NAV_{fund,0} = w_1 P_{1,0} + w_2 P_{2,0} \,, \tag{4}$$

$$\therefore\ NAV_{fund,0} = 0.50 * 800 + 0.50 * 950 = \$875.00 \,.$$

$$NAV_{fund,1} = w_1(PMT_{1,1} + P_{1,1}) + w_2(PMT_{2,1} + P_{2,1}) \,, \tag{8}$$

$$\therefore NAV_{fund,1} = 0.50(0 + 920) + 0.50(70 + 960) = \$975.00 \,.$$

$$NAV_{fund,2} = w_1(PMT_{1,1} + PMT_{1,2} + P_{1,2}) + w_2(PMT_{2,1} + PMT_{2,2} + P_{2,2}) \,, \tag{13}$$

$$\therefore NAV_{fund,2} = 0.50(0 + 0 + 1000) + 0.50(70 + 70 + 1000) = \$1070.00 \,.$$

$$r_{fund,1} = \frac{w_1(PMT_{1,1}+P_{1,1}-P_{1,0})+w_2(PMT_{2,1}+P_{2,1}-P_{2,0})}{w_1 P_{1,0}+w_2 P_{2,0}} \,, \tag{15}$$

$$\therefore r_{fund,1} = \frac{0.50(0 + 920 - 800) + 0.50(70 + 960 - 950)}{(0.50 * 800) + (0.50 * 950)} \approx 11.429\% \,.$$

$$r_{fund,2} = \frac{w_1(PMT_{1,1}+PMT_{1,2}+P_{1,2}-P_{1,0})+w_2(PMT_{2,1}+PMT_{2,2}+P_{2,2}-P_{2,0})}{w_1 P_{1,0}+w_2 P_{2,0}} \,, \tag{16}$$

$$\therefore r_{fund,2} = \frac{0.50(0 + 0 + 1000 - 800) + 0.50(70 + 70 + 1000 - 950)}{(0.50 * 800) + (0.50 * 950)} \approx 22.286\% \,.$$

The application of Model I can be seen through the values stated in Table 4.3, in which the cumulative returns earned on the mutual fund over a period can be calculated using the cumulative coupon payments received and any capital gain/loss incurred due to the change in prices of the bonds over a period.

4.2 Model II

To test Model II in chapter 3, the formulae are applied to the hypothetical mutual fund created in the illustration above. Table 4.4 summarizes the returns earned on both bonds over the two periods and the weighted average bond return earned on the bonds.

Table 4.4: Bond Return Values Calculated Using Model II

Underlying Bonds	Period *0*	Period *1*	Period *2*
Bond *1*	0.00%	15.000%	25.000%
Bond *2*	0.00%	8.421%	20.000%
Weighted Average Bond Return		11.711%	22.500%

The periodic returns on bonds and the weighted average bond returns are calculated in the following manner using formulae (20), (24), (27), and (29) from Model II and data from Table 4.1 and Table 4.2.

$$r_{1,1} = \frac{(1+y_{1,1})^2}{(1+y_{1,2})} - 1\,, \tag{20}$$

$$\therefore r_{1,1} = \frac{(1+0.11803)^2}{(1+0.08696)} - 1 = 15.000\%\,.$$

$$r_{1,2} = (1+y_{1,1})^2 - 1\,, \tag{24}$$

$$\therefore r_{1,2} = (1+0.11803)^2 - 1 = 25.000\%\,.$$

$$r_{2,1} = \frac{1+\frac{1+\frac{1}{CR_2}}{1+y_{2,2}}}{\frac{1}{1+y_{2,1}}+\frac{1+\frac{1}{CR_2}}{(1+y_{2,1})^2}} - 1\,, \tag{27}$$

$$\therefore r_{2,1} = \frac{1+\frac{1+\frac{1}{0.07}}{1+0.11458}}{\frac{1}{1+0.09876}+\frac{1+\frac{1}{0.07}}{(1+0.09876)^2}} - 1 = 8.421\% \,.$$

$$r_{2,2} = \left[\frac{2+\frac{1}{CR_2}}{2+y_{2,1}+\frac{1}{CR_2}} * \left(1+y_{2,1}\right)^2\right] - 1 \,, \tag{29}$$

$$\therefore r_{2,2} = \left[\frac{2+\frac{1}{0.07}}{2+0.09876+\frac{1}{0.07}} * (1+0.11458)^2\right] - 1 = 20.000\% \,.$$

$$(w_1 * r_{1,1}) + (w_2 * r_{2,1}) = (0.50 * 0.15) + (0.50 * 0.08421) = 11.711\% \,,$$

and $$(w_1 * r_{1,2}) + (w_2 * r_{2,2}) = (0.50 * 0.25) + (0.50 * 0.20) = 22.500\% \,.$$

Table 4.5 summarizes the differences between cumulative returns earned on mutual fund stated in Table 4.3 and the weighted average bond returns stated in Table 4.4.

Table 4.5: Difference between Cumulative Return and Weighted Average Bond Return

Period	Cumulative Return ($r_{fund,t}$)	Weighted Average Bond Return	Difference
Period *1*	11.429%	11.711%	-0.282%
Period *2*	22.286%	22.500%	-0.214%

Referring to the differences calculated in Table 4.5, it can be inferred that the relation developed in formulae (30) and (31) is correct, and the weighted average bond returns can approximately predict the cumulative return earned on the mutual fund.

$$r_{fund,1} \approx (w_1 * r_{1,1}) + (w_2 * r_{2,1}) \,, \tag{30}$$

and $$r_{fund,2} \approx (w_1 * r_{1,2}) + (w_2 * r_{2,2}) \,. \tag{31}$$

The following set of data were used to empirically test the accuracy of Model II. Table 4.6 summarizes the results generated in different scenarios by changing the coupon rates and bond prices over period *0* and *1* for bond *1* and bond *2*. All other variables used in Table 4.1 and Table 4.2 are kept constant.

Table 4.6: Scenario Analysis of Model II

Period	Coupon Rate		Market Price		Cumulative Return ($r_{fund,t}$)	Weighted Average Bond Return	Difference
	Bond *1*	Bond *2*	Bond *1*	Bond *2*			
Period *0*			$750.00	$900.00			
Period *1*	5.00%	6.50%	$840.00	$945.00	15.152%	15.444%	-0.293%
Period *2*			$1,000.00	$1,000.00	35.152%	36.111%	-0.960%
Period *0*			$800.00	$880.00			
Period *1*	0.00%	0.00%	$900.00	$930.00	8.929%	9.091%	-0.162%
Period *2*			$1,000.00	$1,000.00	19.048%	19.318%	-0.271%
Period *0*			$980.00	$860.00			
Period *1*	7.50%	0.00%	$1,050.00	$970.00	13.859%	13.793%	0.065%
Period *2*			$1,000.00	$1,000.00	16.848%	16.813%	0.035%

The trivial differences between cumulative fund returns and weighted average bond returns in each scenario imply that there exists an approximation between the two variables and the relation developed in Model II is accurate.

CHAPTER 5

CONCLUSION

Mutual funds are financial instruments that invest in a diversified pool of securities by accumulating investments made by individual investors. This study models the relationship between the cumulative returns on mutual funds and underlying bonds' returns and prices.

A typical mutual fund's return is calculated by determining the change in the NAV over a period. Model I expands this relation by equating the NAV to the weighted average of cumulative coupon payments received and any capital gain/loss incurred due to change in prices of underlying securities over a period. Model II is based on an empirically tested assumption that the sum of the weighted average returns of underlying securities can reasonably approximate the holding mutual fund's return. Further, Model II represents the bond returns in terms of bond yields over each period. It also illustrates the relation between weighted average bond returns and cumulative mutual fund returns by creating a hypothetical fund. This aspect of the study will serve as a primary source for future research endeavors to the best of our knowledge.

The study provides a broad explanation for the relation between mutual fund returns and the returns earned on underlying securities. Academic studies of fixed-income markets typically use data on individual bond prices and returns. Studies on the mutual fund industry almost always analyze fund returns. However, it is rare that studies on fixed-income mutual fund performance are directly linked to the yields, prices, or returns on the underlying bonds that are held. This is partially since data on mutual funds' bond holdings are not as easily available as funds' holdings of stocks, as well as to the fact that the relationship between bond prices and yields, and those of fixed-income mutual funds, is rarely examined in the academic literature. This study will allow for a better approximation between fixed-income fund performance and the performance of underlying holdings. Researchers will then be able to better compare the performance of fixed-income mutual funds with what is known about equity mutual funds.

Finally, future research could potentially expand the models illustrated by changing the bond parameters. One could increase the time-to-maturity for the bonds and change the frequency of coupon payments from annually to semiannually or monthly. The mutual fund could potentially be expanded either by increasing the total number of underlying securities or by incorporating various other securities to diversify the fund. By changing certain variables in the models, the accuracy of the results obtained by this study could be further tested. The level of conformity between such results and the results obtained by this study will be a useful analysis to conduct.

APPENDIX A

COMPUTATION OF BOND RETURNS

Coupon Payments received on a bond remain the same over each period as they are calculated as a product of face value and coupon rate of bond, which remains constant over the life of bond. Thus, $PMT_{1,1} = PMT_{1,2}$.

Scenario 1: Bond *1* and *2* are considered zero-coupon bonds.

The formula for return on bond *1* over period *1*:

$$r_{1,1} = \frac{PMT_{1,1}+P_{1,1}-P_{1,0}}{P_{1,0}}, \tag{A1}$$

$$r_{1,1} = \frac{PMT_{1,1}+P_{1,1}}{P_{1,0}} - \frac{P_{1,0}}{P_{1,0}},$$

$$r_{1,1} = \frac{PMT_{1,1}+P_{1,1}}{P_{1,0}} - 1,$$

and

$$r_{1,1} = \frac{PMT_{1,1}+\frac{PMT_{1,2}}{1+y_{1,2}}+\frac{FV_{1,2}}{1+y_{1,2}}}{\frac{PMT_{1,1}}{1+y_{1,1}}+\frac{PMT_{1,2}}{(1+y_{1,1})^2}+\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1.$$

As the bond is assumed to be zero-coupon there will be no coupon payments.

Thus, $PMT_{1,1} = PMT_{1,2} = 0$, and we have:

$$r_{1,1} = \frac{0+\frac{0}{1+y_{1,2}}+\frac{FV_{1,2}}{1+y_{1,2}}}{\frac{0}{1+y_{1,1}}+\frac{0}{(1+y_{1,1})^2}+\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1,$$

$$r_{1,1} = \frac{\frac{FV_{1,2}}{1+y_{1,2}}}{\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1,$$

$$r_{1,1} = \frac{FV_{1,2}}{1+y_{1,2}} * \frac{(1+y_{1,1})^2}{FV_{1,2}} - 1,$$

and

$$r_{1,1} = \frac{(1+y_{1,1})^2}{1+y_{1,2}} - 1. \tag{A2}$$

The formula for return on bond *1* over period *2* is:

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+P_{1,2}-P_{1,0}}{P_{1,0}}. \tag{A3}$$

As the bond's term is assumed to be for two periods, in period *2* the price of bond is assumed to be equal to the face value. Thus, $P_{1,2} = FV_{1,2}$:

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+FV_{1,2}-P_{1,0}}{P_{1,0}} ,$$

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+FV_{1,2}}{P_{1,0}} - \frac{P_{1,0}}{P_{1,0}} ,$$

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+FV_{1,2}}{P_{1,0}} - 1 ,$$

and

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+FV_{1,2}}{\frac{PMT_{1,1}}{1+y_{1,1}}+\frac{PMT_{1,2}}{(1+y_{1,1})^2}+\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1 .$$

As the bond is assumed to be zero-coupon, there will be no coupon payments.

Thus, $PMT_{1,1} = PMT_{1,2} = 0$, and we have:

$$r_{1,2} = \frac{0+0+FV_{1,2}}{\frac{0}{1+y_{1,1}}+\frac{0}{(1+y_{1,1})^2}+\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1 ,$$

$$r_{1,2} = \frac{FV_{1,2}}{\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1 ,$$

$$r_{1,2} = FV_{1,2} * \frac{(1+y_{1,1})^2}{FV_{1,2}} - 1 ,$$

and

$$r_{1,2} = (1+y_{1,1})^2 - 1 . \tag{A4}$$

Using the same algorithm, returns on bond *2* can be calculated.

The formula for return on bond *2* over period *1* is:

$$r_{2,1} = \frac{(1+y_{2,1})^2}{1+y_{2,2}} - 1 . \tag{A5}$$

The formula for return on bond *2* over period *2* is:

$$r_{2,2} = (1+y_{2,1})^2 - 1 . \tag{A6}$$

Scenario 2: Bond *1* and *2* are considered bonds paying coupon payments at a specified coupon rate (CR_b, where *b* refers to bond number).

The formula for return on bond *1* over period *1* is:

$$r_{1,1} = \frac{PMT_{1,1}+P_{1,1}-P_{1,0}}{P_{1,0}}\,, \tag{A7}$$

$$r_{1,1} = \frac{PMT_{1,1}+P_{1,1}}{P_{1,0}} - \frac{P_{1,0}}{P_{1,0}}\,,$$

$$r_{1,1} = \frac{PMT_{1,1}+P_{1,1}}{P_{1,0}} - 1\,,$$

and
$$r_{1,1} = \frac{PMT_{1,1}+\frac{PMT_{1,2}}{1+y_{1,2}}+\frac{FV_{1,2}}{1+y_{1,2}}}{\frac{PMT_{1,1}}{1+y_{1,1}}+\frac{PMT_{1,2}}{(1+y_{1,1})^2}+\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1\,.$$

As coupon payments remain same over each period, $PMT_{1,1} = PMT_{1,2} = \text{x}$, and we have:

$$r_{1,1} = \frac{x+\frac{x}{1+y_{1,2}}+\frac{FV_{1,2}}{1+y_{1,2}}}{\frac{x}{1+y_{1,1}}+\frac{x}{(1+y_{1,1})^2}+\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1\,,$$

$$r_{1,1} = \frac{\frac{x(1+y_{1,2})+x+FV_{1,2}}{1+y_{1,2}}}{\frac{x(1+y_{1,1})+x+FV_{1,2}}{(1+y_{1,1})^2}} - 1\,,$$

and
$$r_{1,1} = \frac{x[\frac{1+y_{1,2}}{1+y_{1,2}}+\frac{1+\frac{FV_{1,2}}{x}}{1+y_{1,2}}]}{x[\frac{1+y_{1,1}}{(1+y_{1,1})^2}+\frac{1+\frac{FV_{1,2}}{x}}{(1+y_{1,1})^2}]} - 1\,.$$

Since, the coupon rate can be calculated by dividing the coupon payment received by the face value, we have:

$$\frac{FV_{1,2}}{x} = \frac{1}{CR_1}\,,$$

and
$$r_{1,1} = \frac{1+\frac{1+\frac{1}{CR_1}}{1+y_{1,2}}}{\frac{1}{1+y_{1,1}}+\frac{1+\frac{1}{CR_1}}{(1+y_{1,1})^2}} - 1\,. \tag{A8}$$

The formula for return on bond *1* over period *2* is:

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+P_{1,2}-P_{1,0}}{P_{1,0}}\,. \tag{A9}$$

As the bond's term is assumed to be for two periods, in period *2* the price of the bond is assumed to be equal to the face value. Thus, $P_{1,2} = FV_{1,2}$, and we have:

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+FV_{1,2}-P_{1,0}}{P_{1,0}} \, ,$$

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+FV_{1,2}}{P_{1,0}} - \frac{P_{1,0}}{P_{1,0}} \, ,$$

$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+FV_{1,2}}{P_{1,0}} - 1 \, ,$$

and
$$r_{1,2} = \frac{PMT_{1,1}+PMT_{1,2}+FV_{1,2}}{\frac{PMT_{1,1}}{1+y_{1,1}}+\frac{PMT_{1,2}}{(1+y_{1,1})^2}+\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1 \, .$$

As the coupon payments remain same over each period, $PMT_{1,1} = PMT_{1,2} = x$, and we have:

$$r_{1,2} = \frac{x+x+FV_{1,2}}{\frac{x}{1+y_{1,1}}+\frac{x}{(1+y_{1,1})^2}+\frac{FV_{1,2}}{(1+y_{1,1})^2}} - 1 \, ,$$

$$r_{1,2} = \frac{x+x+FV_{1,2}}{\frac{x(1+y_{1,1})+x+FV_{1,2}}{(1+y_{1,1})^2}} - 1 \, ,$$

$$r_{1,2} = \left[\frac{x\left[1+1+\frac{FV_{1,2}}{x}\right]}{x\left[1+y_{1,1}+1+\frac{FV_{1,2}}{x}\right]} * \left(1+y_{1,1}\right)^2\right] - 1 \, ,$$

and
$$r_{1,2} = \left[\frac{2+\frac{1}{CR_1}}{2+y_{1,1}+\frac{1}{CR_1}} * \left(1+y_{1,1}\right)^2\right] - 1 \, . \qquad \text{(A9)}$$

Using the same algorithm, returns on bond *2* can be calculated.

The formula for return on bond *2* over period *1* is:

$$r_{2,1} = \frac{1+\frac{1+\frac{1}{CR_2}}{1+y_{2,2}}}{\frac{1}{1+y_{2,1}}+\frac{1+\frac{1}{CR_2}}{(1+y_{2,1})^2}} - 1 \, . \qquad \text{(A10)}$$

The formula for return on bond *2* over period *2* is:

$$r_{2,2} = \left[\frac{2+\frac{1}{CR_2}}{2+y_{2,1}+\frac{1}{CR_2}} * \left(1+y_{2,1}\right)^2\right] - 1 \, . \qquad \text{(A11)}$$

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BIOGRAPHICAL INFORMATION

Sanjari Chelawat is graduating with an Honors Bachelor of Business Administration in Accounting and Finance in May 2018, and is a candidate for *Summa Cum Laude* Latin Honors. After completing her undergraduate studies, Sanjari plans to pursue a Master of Science in Accounting at the University of Texas at Arlington (UTA) and eventually become a Certified Public Accountant (CPA). She is a recipient of the Honors College Undergraduate Research Fellowship, the UTA Asia Scholarship, the Freshman Honors Scholarship, and has served as an Honors Advocate for two years. Sanjari's research interests include modelling relationships between mutual funds and underlying securities and corporate governance issues in companies.