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# CALCULUS STUDENTS’ PROBLEM-SOLVING STRATEGIES ON RELATED RATES OF CHANGE PROBLEMS APPEARING IN ONLINE VERSUS PAPER-AND-PENCIL FORMAT

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# CALCULUS STUDENTS' PROBLEM-SOLVING STRATEGIES ON RELATED RATES OF CHANGE PROBLEMS APPEARING IN ONLINE VERSUS PAPER-AND-PENCIL FORMAT

by

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Abstract

CALCULUS STUDENTS' PROBLEM-SOLVING STRATEGIES ON RELATED RATES OF CHANGE PROBLEMS APPEARING IN ONLINE VERSUS PAPER-AND-PENCIL FORMAT

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The University of Texas at Arlington, 2024

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This study explores first-semester calculus students' use of mathematical problem-solving strategies while working related rates of change problems in both an online homework format and a traditional pencil-paper format. We address two research questions: (1) How do students' mathematical problem-solving strategies when working online homework on related rates of change problems compare with their problem-solving strategies when working paper-and-pencil homework related rates of change problems? (2) What influence does the 'view an example' feature in online homework have on a student's problem-solving strategies when working an online RRC homework problem? Using scores on free-response midterm exam problems on related rates of change to select participants, we then conducted task-based interviews in which participants were asked to solve four (two paper-pencil and two online) related rates of change problems. Thematic analysis methods (Braun & Clarke, 2006) were used to analyze interview data recordings, transcriptions, and student work. In this setting, we identified more instances of problem-solving strategy use when participants engaged in the paper-pencil format related rates of change problems. In addition, instances of problem-solving strategy use by participants from the highest midterm score quartiles were higher than for those with scores in other quartiles represented. When participants used the ‘view an example’ online homework feature, fewer instances of problem-solving strategy use were identified. Four uses of the ‘view an example’ feature emerged from participant data: use to mimic, use to learn the process, use for sense-making, and non-use of the feature. The findings suggest that participants may be using features of online homework platforms on related rates of change problems in a manner that circumvents opportunities to engage in mathematical problem solving.

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# Chapter 1

# Introduction

"Mathematics isn't just about mastering facts and procedures, but that it's also about asking questions (problem posing, if you will) and then pursuing the answers in a reasoned ways" (Schoenfeld, 2013, p.27). Polya's (1957) early work in mathematical problem-solving is often used to describe the problem-solving process, but "definitions of mathematical problem solving or what constitutes a mathematics problem may vary widely" (Álvarez et al., 2018, p. 233). However, the definition posed by Lester that "…a problem is an ask for which an individual does not know (immediately) how to get an answer…" (Lester, 2013, as cited in Álvarez et al., 2018, p. 233) grounds the idea of problem and problem solving relative to the solver and may be most salient for considering how modern day interactions with online homework problems reposition the problem solver. This is an area for which little prior research exists.

Ellis et al. (2015) found that student's growth and confidence in a calculus course can be attributed to two components of an online homework system: (1) multiple opportunities to solve a problem correctly, and (2) the ability to receive instant feedback. In addition, with the rapid advancement of supporting technology, the implementation of online homework has become more widespread in recent years. Furthermore, the uptake of these platforms also accelerated due to the precautions taken worldwide due to the COVID-19 pandemic (Telli et al., 2023).

With the availability of many online platforms for assigning and grading homework, more and more of the homework in calculus no longer entails paper-and-pencil homework (Dorko, 2020a). As such, with the prevalence of associated scaffolding embedded in these platforms, more needs to be understood about whether this tends to over-proceduralize topics, such as related rates of change, and undermine roles these topics may have in further developing students' capacity in mathematical problem solving or mathematical reasoning. Engelke (2008) maintains that to successfully solve related rates of change problems, students require well-developed mathematical problem-solving skills. However, Mkhatshwa (2019) asserts that few studies have explored students' reasoning on solving related rates of change problems. The purpose of this investigation is to explore how problem-solving strategies may differ when solving related rates of change problems presented in a traditional paper-and-pencil format versus related rates of change problems presented in an online platform which includes typical options for scaffolding help as well as "view an example" features.

Calculus is a requirement for all STEM (science, technology, engineering, and mathematics) majors (Carlson et al., 2010; Carlson et al., 2015; Sadler & Sonnert, 2017, 2018; Schraeder et al., 2019). In a first-semester calculus course, students are taught the concept of related rates of change, problems that involve at least two 'rate' quantities that can be related by an equation, function, or formula, and these problems tend to be challenging for most students in differential calculus (Engelke, 2004; Engelke Infante, 2021; Martin, 2000; Mkhatshwa, 2018, 2019, 2020). In addition, researchers find that student difficulties with solving related rates of change problems involve issues with conceptual understanding of the mathematics (Engelke, 2004; Martin, 2000; Mkhatshwa, 2018, 2019, 2020).

Currently, online homework offers convenience, accessibility, immediate feedback, and ease of tracking student progress. As such, related rates of change problems also appear as homework problems in the online platforms. In addition, as class sizes in introductory courses increase and features of online homework platforms become more responsive, online homework is an increasingly popular tool for teaching and learning mathematics (Archer, 2018). Since first-semester calculus students at

United States institutions spend more time doing homework than they do in a Calculus 1 classroom (Ellis et al., 2015; Dorko, 2020a, 2020b, 2021), online homework gives students some support systems such as the ability to have multiple attempts to solve problems, the ability to use 'see another example', and the ability to use 'help me solve' buttons that are located within the online homework systems. Moreover, the use of multiple attempts offered by online homework has been shown to improve student learning (Archer, 2018). In addition, the use of buttons like 'practice another version' have been used to troubleshoot mistakes and to use as a template to solve online homework problems (Dorko, 2020a).

Few would dispute that solving related rates of change problems involves mathematical problem-solving and problem-solving strategies. Problem-solving is often described as the process employed when solving a problem that is novel and unfamiliar (Carlson & Bloom, 2005; Álvarez et al., 2019; Dawkins & Epperson, 2014). With the support systems that are offered in online homework that are not available with textbook paper-and-pencil homework, problem-solving strategies used by students when working in different platforms may differ. This investigation explores how problem-solving strategies may differ when solving related rates of change online homework problems with the availability of supports versus textbook paper-and-pencil related rates of change problems. The following research questions are addressed:

(1) How do students' problem-solving strategies when working online homework on related rates of change problems compare with their problem-solving strategies when working paper-and-pencil homework related rates of change problems?

(2) What influence does the "view an example" feature in online homework have on a student's problem-solving strategies when working an online related rates of change homework problem?

Very few studies have explored students' reasoning on solving related rates of change problems (Mkhatshwa, 2019). In addition, there is limited research examining student understanding of and student solution strategies for related rates of change problems in Calculus 1 (Engelke, 2008). Likewise, there is little to no research investigating the use of problem-solving strategies when solving related rates of change problems online in comparison to solving related rates of change problems in non-online settings.

In this study, a mixed methods theory is used to formulate a theory for characterizing the mathematical problem-solving of students in Calculus 1. Quantitative data generated from student work on midterms is used in the selection of participants for the task-based interviews. Qualitative methods and thematic analysis (Braun & Clark, 2006) is chosen to explore individuals' problem-solving strategies in-depth while trying to identify common themes among the cases. Fourteen participants agreed to take part in an audio-video recorded task-based interviews where they were asked to solve four, two online and two paper-and-pencil, related rates of change homework problems. A priori codes used for coding were based on a blend of Álvarez et al.'s (2019) characteristics or domains of problem-solving and Carlson and Bloom's (2005) Multidimensional Problem-Solving Framework. Emergent codes were identified using open coding techniques (Braun & Clarke, 2006).

In this setting, we identified more instances of problem-solving strategy use when participants engaged in the paper-pencil format related rates of change problems. In addition, instances of problem-solving strategy use by participants from the highest midterm score quartiles were higher than for those with scores in other quartiles represented. When participants used the 'view an example' online homework feature, fewer instances of problem-solving strategy use were identified. Four uses of the 'view an example' feature emerged as patterns: use to mimic, use to learn the process, use to sense-make, and non-use of the feature. The findings suggest that participants may be using features of online homework platforms on related rates of change problems in a manner that circumvents opportunities to use mathematical problem-solving strategies.

# Chapter 2

# Literature Review

## 2.1 Introduction

This study investigates calculus students' problem-solving strategies when working related rates of change problems appearing in online homework versus similar problems in a paper-and-pencil format. This literature review focuses on five areas salient for this study. To examine calculus students' problem-solving strategies, research on foundational preparation for calculus and mathematical problem solving as well as research on student learning to solve related rates of change problems form three of the five critical areas from the research literature. Since the study also focuses on the interaction with online homework and learning or knowledge transfer from one platform to another, research on the affordances with the use of online homework and learning transfer is also pertinent to this study.

## 2.2 Foundational Preparation for Calculus

Frank and Thompson (2021) investigated students' transition from pre-calculus to calculus. The authors assert that students' readiness to develop conceptual understandings of key ideas in calculus depends significantly on their development of important meanings in their early schooling (Frank & Thompson, 2021). The researchers evaluated students' prior schooling to understand student difficulties in calculus. They found students have limited opportunities to construct mathematical meanings productive for understanding calculus. Also, there is a disconnect between meanings conveyed by textbooks and held by teachers and meanings that would be productive for students' understanding of major ideas in calculus (Frank & Thompson, 2021). The researchers define 'understanding' as a cognitive state resulting from assimilation and a 'meaning' as the space of implications of an understanding (Frank & Thompson, 2021). The findings suggest that teachers and students, in the United States, "share many 'meanings' for slope, average rate of change, and function notation, and these 'meanings' are unproductive for understanding calculus" (Frank & Thompson, 2021, p.561). For example, a productive meaning of average rate of change would include interpreting it as the constant rate of change needed to produce the same net change in the dependent variable for a specified change in the independent variable and not as an arithmetic mean which is unproductive when learning calculus (Frank & Thompson, 2021). That is, when a teacher lacks conceptual 'understanding' of these concepts the unproductive 'meanings' are passed to their students.

In a similar study, White and Mitchelmore (1996) investigated the conceptual knowledge of students in an Introductory Calculus I course who had taken calculus in secondary school. White and Mitchelmore's goal was to "investigate the performance on calculus application problems of a group of students who had previously experienced a traditional introductory calculus course and, thereby, to infer the role of their conceptual knowledge (or lack of it) in solving application problems" (White & Mitchelmore, 1996, p. 80).  The researchers were looking for a correlation between taking a high school calculus course and understanding the concept of a variable. The findings were that a major source of students' trouble in applying calculus is rooted in an underdeveloped concept of a variable and not using variables as quantities that can be related which is similar to the findings of Frank and Thompson's (2021) study. The research revealed that students frequently use variables as items to be manipulated instead of quantities to be related. Students were found to have failed to differentiate a general relationship from a specific value, searching for symbols to apply known procedures without regard to what the symbols refer to, and remembering procedures only in terms of the symbols used when they were first learned (White & Mitchelmore, 1996). The authors defined abstract-apart as “students showing the manipulation focus have a concept of a variable that is limited to algebraic symbols; they have learned to operate with symbols without any regard to their possible contextual meaning” (White & Mitchelmore, 1996). The authors concluded that abstract-apart ideas are simpler to grasp because they are limited to a purely symbolic context. Also, the abstract-apart concept might be sufficient to deal with routine procedures, but the limitations of solely procedural knowledge become obvious when the symbols have a specific contextual meaning. White and Mitchelmore (1996) defined concepts that are formed in the sequence of generalizing, synthesizing, and abstracting as abstract-general. The authors suggest that learning abstract-general concepts requires the formation of links among a wide variety of superficially different contexts and that this is more intellectually demanding. The abstract-general concepts lead to contexts where there are no visible cues, and the learned relationships can then be used to solve more diverse problems.

White and Mitchelmore (1996) suggested that an abstract-general concept of a variable, at or near where a student can create variables to solve complex problems, as a prerequisite to being a successful student in Calculus I. The researchers also suggest that it is not realistic to attempt to provide remedial activities during the calculus course for those students who have an abstract-apart concept of a variable. The authors concluded that entrance requirements must be more stringent in terms of variable understanding, or an appropriate precalculus course should be offered at the university level.

There have also been studies to identify the effects of secondary mathematics courses on student readiness to transition to post-secondary mathematics courses (Frank & Thompson, 2021; Schraeder et al., 2019; White and Mitchelmore, 1996). Sadler and Sonnert’s (2018) study gave empirical evidence that addresses college professors, high school teachers, mathematics researchers, and students' beliefs about the value of high school preparatory courses in mathematics for students who later enrolled in college calculus. The study found that on average, students who had higher grades in prior mathematics courses or higher ACT or SAT scores earned higher grades in college calculus. The data revealed that performance in earlier pre-college courses persisted as a predictor, even how a student performed in Algebra 1 five to six years earlier, on how a student would perform in a college calculus course. Sadler and Sonnert (2018) identified some limitations to this study. The first limitation is that this study was not experimental. Another limitation of this study is that students' studying habits, applying effort, feelings of anxiety about mathematics, tutoring, belonging or not belonging to a study group, connecting with their professor, and varying non-academic issues may have contributed to two-thirds of the variance left unexplained by this study. There was also the limitation of the sample being limited to those taking college calculus.

Schraeder, Pyzdrowski, and Miller (2019) specifically examined the impact of having prior exposure to calculus on students' grades in a college Calculus I course. The entry method that a student took to take Calculus 1 was a departmental placement test or pre-requisite course. The entry method made a significant difference in both whether a student passed Calculus I and the students' overall letter grade in Calculus I. Students who were placed into Calculus I via some placement criteria performed better than those who took the pre-requisite classes. The researchers found two potential reasons for the inconsistency in student achievement. The first one being that the pre-requisite courses were inadequate. The second potential reason being the disparity in ability between the two groups of students and placement criteria has gradually increased at the university. The implication is that students who test directly into Calculus I are better prepared mathematically than those who must take pre-requisite classes. This results in the more capable students having greater success in the more difficult courses. An overwhelming majority of students felt that having prior exposure to calculus before taking Calculus 1 was an advantage over not having any exposure to calculus before taking a Calculus 1 course. None of the participants in the study felt that prior exposure to calculus was necessary to pass Calculus I. The researchers state that a limitation of this study was that a vast majority of the students who completed the survey passed the class and the opinion of those who failed the course may not have been fully exposed. There also were three subgroups that were not interviewed.

While there are many studies about the prerequisite skills needed to be successful in a calculus course, research by Carlson, Oehrtman, and Engelke (2010) to assess those prerequisite skills provides more ways to think about the classification of these skills.   Carlson, et al. used what is known about foundational reasoning abilities for calculus to develop the *Precalculus Concept Assessment*. The Precalculus Concept Assessment instrument is a 25 multiple-choice exam that was designed to: assess student learning in college algebra and precalculus, compare the effectiveness of different curriculum designs, and determine student readiness for calculus. Although the purpose of a precalculus course should be to prepare students for a calculus course, Carlson et al. (2010) observe that fewer than half of the students that complete a precalculus course successfully enroll in a calculus course. As such, there is limited research firmly establishing the conceptual foundations necessary for success in a calculus course.

Although Carlson et al. (2010) recognize that more research still needs to be conducted to derive a comprehensive list of understandings needed for calculus, they aimed was to develop a tool to assess *essential* knowledge that is fundamental for student achievement and comprehension of fundamental concepts of a first-semester calculus course. The authors state that being taught procedures without focusing on developing coherent understanding at the same time is not effective in developing conceptions that support meaningful interpretation and use of concepts when solving new problems (Carlson et al., 2010).

In precalculus and introductory calculus courses, Carlson et al. (2010) discuss the difficulty of learning and understanding the concept of function. The researchers assert that concept of function is “the central conceptual strand of mathematics curriculum” in lower-level mathematics (Carlson et al., 2010). The researchers reference previous research on pre-calculus students’ having a static image of the function concept. This fixed view of a function can lead to an action view of a function. “An action view of function is described as when students tend to view functions only in terms of symbolic manipulations and procedural techniques disassociated from the underlying interpretation of function as a more general mapping of a set of input values to a set of output values” (Breidenbach et al.,1992; Dubinsky & Harel, 1992; Carlson, 1998, as cited in Carlson et al., 2010, p. 115). A student can progress from an action view of a function into a process view of a function. A process view of a function is when “students can imagine a continuum of input values in the domain of a function resulting in a continuum of output values” (Breidenbach et al.,1992; Dubinsky & Harel, 1992; Carlson, 1998, as cited in Carlson et al. 2010).

Carlson et al. (2010) also assert that a student can gain a better understanding of the major concepts in a calculus course when they can use covariational reasoning to interpret the dynamics of quantities in function situations. “Covariational reasoning is when a student can interpret the meaning of a function modelling a dynamic situation also requires the attention to how the output values of a function are changing while imagining changes in a function's input values" (Carlson et al., 2010, p.115). Further, a process view of function is necessary in interpreting the meaning of a graph, computational reasoning, and function and covariational reasoning to be prepared for a calculus course.

Later in 2015, Carlson, Madison, and West examined students' readiness to learn calculus. The authors state that prior research has shown that students are not being prepared to be successful in a calculus course (Breidenbach et al., 1992; Carlson, 1998; Moore, 2012; Moore & Carlson, 2012, as cited by Carlson et. al., 2015). The Calculus Concept Readiness Taxonomy (CCR) was developed to identify precalculus reasoning and understanding needed for learning key concepts in calculus. From this, Carlson et al. (2015) found that calculus students had severe weaknesses in fundamental knowledge and the thought processes to learn calculus. The majority of students from the study were unable to correctly solve proportional reasoning questions and the three function word problems. Students also had trouble with the composition of functions and constructing meaningful formulas by examining the quantities in a dynamic word problem context. Carlson et al. (2015) suggest that there is a need for higher standards for curriculum and courses prior to calculus in terms of the degree to which they support students' development of fundamental reasoning abilities and understandings needed for learning and using central ideas of calculus (Carlson et al., 2015, p. 229).

According to several research studies, students' difficulties in calculus can mostly be attributed to a limited conceptual understanding of variable, function, slope, and rate of change—all ideas developed in school mathematics (Thompson & Harel, 2021; Larsen et al., 2017). These concepts are all foundational for understanding the ideas of calculus (Thompson & Harel, 2021). Additionally, understanding functions covariationally, as an invariant relationship between two quantities' values as they vary at the same time, is the most important meaning of function for students learning calculus (Thompson & Harel, 2021; Madison et al., 2015; Frank & Thompson, 2021). As such, when learning to solve related rates of change problems, covariational reasoning plays a prominent role in this process (Engelke, 2004; Engelke Infante, 2021; Mkhatshwa, 2019, 2020).

## 2.3 Online Homework

Many lower-level undergraduate mathematics courses utilize both paper-and-pencil and online homework. The use of online homework holds optimism for supporting student learning because the platforms provide multiple attempts, immediate feedback, hints, and other ‘help’ features (Dorko, 2021). However, the effectiveness of online homework on student learning, promoting student learning, and student engagement are largely unknown. In recent years, investigations have been conducted on online homework and student engagement and how this engagement affects student achievement.

Ellis et al. (2015) analyzed the characteristics of Calculus 1 homework given in a successful mathematics program compared to a mathematics program that is not considered as successful. Student success was measured by increased confidence, grade in Calculus 1, interest, enjoyment of mathematics, and enrollment in Calculus 2. The study used a mixed method analysis and used the Characteristics of Successful Programs in College Calculus (CSPCC) project (Ellis et al., 2015). The CSPCC study involved 500 institutions and involved two phases. The first phase was a survey given to Calculus I students and their instructors at the beginning and end of the Fall semester. The second phase of the CSPCC study included an explanatory case study at five Ph.D. granting universities that had more successful Calculus I programs as measured by the factors given earlier. The case study data was used to understand the nature of the homework assignments at selected and non-selected universities. Related rates of change homework and exam problems were examined to better understand the nature of the problems assigned at these institutions. Inductive thematic analysis was conducted on the focus group interviews. Thematic analysis is defined by the authors as identifying, analyzing, and reporting themes within the data.

Ellis et al. (2015) found three salient features related to the nature of the homework used in successful Calculus I programs: structure, content, and feedback. *Structure* refers to the format and delivery mode of homework, the frequency of homework, the amount of homework, the level of coordination by instructors, whether it was online or written, and if it was individual or group homework. *Content* refers to the nature of the homework or tasks on the homework, whether they emphasize or assess procedural or conceptual knowledge. *Feedback* refers to dialogue between the instructor and the student that informs the student about the quality of his or her work. The structure that universities with more successful Calculus I programs utilized included assigning homework and group projects more often and more frequent use of online homework systems. The content of these successful universities focused on incorporating more novel and cognitively demanding tasks in the course. The feedback given by universities with successful Calculus 1 programs described homework practices in which homework was assigned more frequently and graded and returned with feedback. The authors suggested that homework must be purposeful about how to utilize online and/or written homework as a medium for students to practice skills and grapple with concepts while providing feedback for the successful development of both.

Whereas Ellis et al. (2015) found that universities with successful Calculus 1 programs assigned homework more often and used online homework, Archer's (2018) study aimed to understand the effects of specific features of online homework on student achievement. Archer (2018) explored the practice of allowing multiple attempts on homework and student learning. Proponents of multiple attempts on homework assert that multiple attempts allow more practice and benefits student learning with the allowance of reworking of problems (Palocsay & Stevens, 2008; Titard et al., 2014, as cited by Archer, 2018). Opponents of multiple attempts on homework maintain that it inflates grades, promotes guessing, and does not promote mastery of the material (Fish, 2015; Rhodes & Sarbaum, 2015, as cited by Archer, 2018). Archer's (2018) study found that student learning increased, based on exam performance, with the web-based system that allowed multiple attempts. It was also found that contrary to some previous studies (Bowman et al., 2014; Fatemi et al., 2015; Rhodes & Sarbaum, 2015, as cited in Archer, 2018), there was no evidence of grade inflation with the allowance of multiple attempts on the homework. Much like Ellis et al.'s (2015) findings about feedback on homework, Archer asserts that the feedback timing of the homework is a contributing factor to the variation of scores.

While the studies Archer (2018) and Ellis et al. (2015) find that timely feedback improved student achievement, Dorko (2020b) analyzes how students engage with online homework. Dorko develops an empirically based model of students' activity as they complete online homework. The empirical model of the actions in which students engage in before submitting an answer entails:

-Students do scratchwork, do calculations on calculator

-Students look at class notes, textbook, supplemental materials

-Students work step of similar problem from textbook, supplemental materials

-Student reasons about what she is 'supposed to learn'

-Student says 'had no idea, I guessed' (Dorko, 2020b, p. 463).

This model developed by Dorko by observation of participants in her study is not mutually exclusive, nor in sequential order, and is not all-inclusive for every participant.

Dorko (2020b) used constant comparative method (Strauss & Corbin, 1994, as cited in Dorko, 2020b) to analyze the data. Dorko's (2020b) study produced three results. The first result is the model (Dorko, 2020b, p.463) describing the nature of students' activity when doing homework in an online homework system. From the model the other two results are presented; the second result is students' activity when solving online homework problems is cyclic and similar to what a mathematician would do when problem-solving. The third result is that students leverage their multiple tries per question and ability to submit parts of questions individually to obtain intermediate feedback which is used to guide their work on the rest of the problem.

Continuing her research about online homework, Dorko (2021) sought to see what students learn from online homework by investigating the student use of 'see similar example' when doing online homework and the impact it had on their learning goals. A previous study conducted by Dorko (2020a) produced results that showed that online homework has the same or slightly better effects on student achievement. Dorko's (2021) study builds on previous literature by providing a detailed investigation into how students employ similar example features; analyzing observational and interview data to complement self-report data; examining links between students' use of similar examples and their goals for that resource use; and adding to the literature about professors' goal for homework by including the course coordinator's goals for homework. A modified instructional triangle was also employed in this study. An instructional triangle defines instruction as the "interactions among teachers and students around content, in environments" (Cohen et al., 2003, p.122, as cited in Dorko, 2020b). In conjunction with the use of a modified instructional triangle, the researcher employed a didactic contract for the shared expectations of the teacher and student. The didactic contract is defined as a set of reciprocal obligations and mutual expectations that is the result of an often-implicit negotiation (Dorko, 2020b, p. 452). The didactic contract for this study is the teacher's expectations of the student to do the assignment and utilize resources as needed. The didactic contract also includes the students' interactions with the assignment and the student's goal and expectations for the provided learning opportunities.

Dorko (2021) found that students used the 'practice another version' problems: to copy and paste the 'practice another version' solution; to view as a template; to see solutions to similar problems to troubleshoot; to check if they were on the right track; to see the steps to solve a problem; and to see the form of the answer. Students' motivation for using 'practice another version' ranged from completing the assignment and getting a good grade to understanding the content. The researcher suggested that students can use 'practice another version' problems more productively and increase student engagement on non-procedural problems. Another suggestion from the author is that instructors need to be more explicit about the reason why they assign various problems so that students can understand their value instead of just completing the assignment for a good grade. The limitations stated by the author included the fact that the online homework problems were procedural, only involved Calculus I students, and the small number of students involved in the study. The author suggests that for future research giving students problems that gave hints and not giving students fully worked out solutions. Another suggestion for future research from the author is an exploration of the connections between students' homework learning when working in small groups. The author suggests that future research could also focus on exploring connections between work on homework problems and their activity on subsequent tasks in homework and other milieu.

As far as preference, students prefer online homework to paper-and-pencil homework and they believe that online homework is more beneficial to their learning, but they also like to have both types in their course (Ellis et al., 2015). A feature of online homework is the option of allowing students multiple attempts on problems. Students like having multiple attempts per problem and use those multiple opportunities when available (Ellis et al., 2015; Dorko, 2018, 2020a). These multiple opportunities benefit students with increased exam grades (Archer 2018). Although multiple attempts on problems have led to better student engagement, it also may increase students guessing answers to problems (Dorko, 2018, 2020a). This contradicts findings that have shown that multiple attempts do not increase guessing behaviors (Archer, 2018). Another feature that is available with online homework is students are given immediate feedback. Students like the immediate feedback that comes with online homework and immediate feedback is associated with increased student learning (Ellis et al., 2015; Archer, 2018). Ellis et al. (2015) found that the one factor that distinguished more successful calculus programs from less successful calculus programs was their homework systems and that the more successful programs were considerably more likely to assign online homework than the less successful calculus programs.

## 2.4 Problem Solving

Schoenfeld (2013) reflected on problem-solving twenty-five years after his Mathematical Problem-Solving (1985) book was published. The book gave a theoretical rationale for his mathematical problem-solving course and evidence that the course improved student mathematical problem-solving. Schoenfeld's book was a framework for the analysis of the success and failure of problem-solving in mathematics and hypothetically in all problem-solving domains. Problem-solving was defined by Schoenfeld (2013) as trying to achieve some outcome when there was no known method to achieve it. He believed that his book gave a framework for analyzing the success or failure of problem-solving potentially in all problem-solving domains.  Schoenfeld ultimate theoretical goal was to provide a theoretical explanation that characterizes every decision made by a problem solver while working on a problem in knowledge-intensive, highly social, goal-oriented activities.

Schoenfeld (2013) wrote that his students became more effective problem solvers and the book provided evidence that his course was effective. After taking his problem-solving course, his students were able to master a range of problem-solving heuristics, they were more effective at monitoring and self-regulation, and had better beliefs about themselves and their mathematical abilities. The book offered a methodological blueprint for developing problem solving instruction.

In Schoenfeld's 2010 book, "How We Think", builds on his earlier work and gives the structure of a general theory of in-the-moment decision making. He gives the evolution of the 1985 framework categories. The newer framework is;

a) The goals the individual is trying to achieve;

b) The individual's knowledge (and more broadly, the resources at his or her disposal);

c) The individual's beliefs and orientations (about himself and the domain in which he or she is working); and

d) The individual's decision-making mechanism (Schoenfeld, p.17)

This new framework integrates access to and implementation of heuristic strategies into the category of knowledge.

One study on problem-solving and calculus was conducted by Dawkins and Epperson (2014). The researchers sought further insight into how calculus instruction promotes students' abilities in mathematical problem-solving. They also wanted to examine problem-solving behaviors and how they relate to student success or failure in a first-semester calculus course. The researchers found that student problem-solving as measured by the planning and executing phase and the checking phase scores improved by a statistically significant amount over the course of the semester. Findings also suggest that students who received grades of A's and B's displayed problem-solving behaviors that showed stronger procedural understanding. These students were also more likely to use reliable problem-solving approaches than the other participants. Students that withdrew from the course showed less fluency with algebraic skills and graphical representations. Students that withdrew also showed less initial knowledge of calculus concepts and algorithms as opposed to the students that earned grades of A's and B's which showed greater initial knowledge of algebraic skills and graphical representations. The researchers found that blended traditional/reform calculus instruction did improve students' problem-solving strategy and accuracy scores and their conclusions and justification scores on the problem-solving assessment used in the study. The data showed that the top-performing students had better algebraic fluency. The researchers found that algebraic fluency was needed to be successful in calculus and that a large amount of calculus failure was related to impoverished understandings of precalculus concepts and skills.

Dawkins and Epperson (2015) state that instructors teach in ways that give preference to algebraic and numerical methods in problem solving which, as a result, channels students' problem-solving heuristics away from more conceptual or visual approaches. The researchers also believe that effort is still needed to achieve the calculus reform/NCTM standards vision for a concept-driven, multi-representational pre-calculus and calculus sequence. The researchers suggested that the data shows that covariational reasoning is not being required or rewarded. The researchers presented three messages for secondary instruction: students need to have a strong concept of basic proficiency in algebraic and graphical registers, top-performing students rely too much on algebraic methods, and learning calculus concepts and procedures adequately in secondary school students will be more successful in tertiary calculus.

In an effort to develop a machine-scorable instrument that gives insight into students' use of mathematical problem solving, Álvarez et al. (2019) built upon prior work that classified the ways in which mathematical problem solving is characterized in the research literature. These characterizations or domains were used to identify ways in which mathematical problem solving might manifest itself in student engagement with mathematical tasks. The mathematical problem-solving domains refined for their study were sense-making, representing/connecting, reviewing, justifying, and challenge. The mathematical problem-solving domains were defined as:

> **Sense-making:** Identifying key ideas and concepts to understand the underlying nature of the problem. Attending to the meaning of the problem used.
>
> **Representing/connecting:** Reformulating the problem by using a representation not already used in the problem or connecting the problem to seemingly disjoint prior knowledge. Using multiple representations or connecting several areas of mathematics (e.g., geometric and algebraic concepts).
>
> **Reviewing:** Self-monitoring or assessing progress as problem solving occurs, or assessing the problem solution (e.g., checking for reasonableness) once the problem-solving process has concluded.
>
> **Justifying:** Communicating reasons for the methods and techniques used to arrive at a solution. Justifying solution method(s) or approach(es).
>
> **Challenge:** The problem must be challenging enough from the perspective of the problem solver to engage them in deep thinking or processes toward a goal (Álvarez et al., 2019, p.235).

The findings indicate that calculus students used the mathematical problem-solving domains with greater frequency which was possibly due to the more advanced content knowledge of the calculus students when compared to students in College Algebra, for example. Álvarez et al. found that participants in Calculus scored statistically higher in sense-making than unsuccessful participants in Calculus. The researchers list a major limitation of this study rests in the fact that Likert-style items linked to mathematical problem-solving do not provide information on social or affective components that may impact problem-solving proficiency.

Carlson and Bloom (2005) sought to build on existing research on mathematics problem-solving. Their investigation focused on gaining new information about the interaction of major aspects of problem-solving that have been identified as important for problem-solving success. A framework was developed by Carlson and Bloom (2005) that used a grounded approach with an open coding technique that was named Multidimensional Problem-Solving Framework. The framework describes how resources and heuristics interacted with the general behaviors exhibited during the four problem-solving phases: orienting, planning, executing, and checking. The framework also characterizes how monitoring and affect were expressed during each of the phases. The Multidimensional Problem-Solving Framework shows the cyclic nature of the problem-solving process and the points at which strategic control influenced a mathematicians' problem-solving decisions and actions. The orientation phase of the framework is when the mathematicians engage in great efforts to make sense of the information presented in the problem. The mathematicians also access their concepts, facts, and algorithms as needed to represent the problem situation. In the framework's planning phase, mathematicians access conceptual knowledge and heuristics to build and assess their conjectures. The execution phase of the framework is where mathematicians do their computations formed in the planning phase. The mathematicians were more efficient and effective during the execution phase when they were more fluent in using varying heuristics, algorithms, and computational approaches. During the checking phase of the framework, the mathematicians verify the reasonableness of their work.

## 2.5 Related Rates

When focusing attention to learning how to solve related rates of change problems, the research focuses on how students set-up and solve related rates of change problems. Several studies focus on conceptual and procedural knowledge necessary for solving related rates of change problems (Mkhatshwa & Jones, 2018; Mkhatshwa, 2019, 2020; Engelke, 2004, 2008; Martin, 2000). Mkhatshwa (2019) investigated calculus students' quantitative reasoning in the context of solving related rates of change problems. There had been little research on students' reasoning about related rates problems (Mkhatshwa, 2019). Mkhatshwa and Jones (2018) study found that students improperly applied the product rule of differentiation and found that this limited their success in related rates problems. Mkhatshwa states that more research is needed about how different modes of reasoning, specifically quantitative reasoning, relate to solving related rates problems. Mkhatshwa (2019) study explicitly examines students' quantitative reasoning in geometric and non-geometric related rates problems.  There were five findings from this study. Mkhatshwa's first three findings were new contributions to the research of students' reasoning about related rates problems. The first finding that is contrary to several studies that have reported students' difficulties on utilizing graphs to connect relationship and quantities this study presents students' perspectives on the usefulness of graphs in making sense of quantities and relationships between quantities in the context of solving related rates problems (Carlson et al., 2002; Johnson, 2012, 2015a, 2015b, as cited by Mkhatshwa, 2019). The author suggests that calculus instructors use diagrams when teaching related rates problems to expand student's quantitative reasoning skills. The second finding is that students have difficulty defining time as a variable when time is the underlying independent variable in related rates of change problems. Six of the students in this study reasoned implicitly about the quantity of time (Mkhatshwa, 2019). This finding was contrary to a study that was done by S.R. Jones in 2017. The third finding is contrary to the findings of previous research that students have difficulty understanding routine geometric related rates problems (Martin, 2000; White & Mitchelmore, 1996). Mkhatshwa's (2019) study found that high-performing students were able to mathematize related rates problems. The fourth finding is some students had difficulty implicitly differentiating the equation PV=kT using the product or quotient rule. This finding was like the findings of Mkhatshwa and Jones (2018) that students lack proficiency with the rules of differentiation. The fifth finding was that one student confused a "rate" quantity of speed with an "amount" quantity of distance. Eight students confused "rate" quantities with other "rate" quantities (e.g., rate of change for the radius and rate of change of the area). Previous research has shown that students tend to confuse 'rate' quantities with 'amount' quantities (Arleback et al., 2013; Lobato et al., 2012; Mkhatshwa & Doerr, 2018b; Prince et al., 2012, as cited by Mkhatshwa, 2019).

Mkhatshwa stated that there were three limitations of the study that students were more successful in solving geometric problems because of their previous exposure to geometric problems, that students were not given a chance to mathematize related rates problems in unfamiliar contexts, and that the underlying independent variable in the tasks were time. The author suggests future research to examine students' ability to mathematize related rates problems when solving non-routine related rates problems and research and that there is a need for research that examines students' quantitative reasoning about related rates problems for which the underlying independent variable is not time.

Building on his previous study, Mkhatshwa (2020) analyzed student reported difficulties when solving related rates problems. The study investigated students' difficulties with solving related rates of change problems from a student perspective. The goal for this study was to identify calculus students' difficulties when engaged in reasoning quantitatively about solving related rates problems. The first finding was that students had difficulty when implicit differentiation was required in related rates problems. The second finding was that the number of variables involved in the related rates of change problem affected students' ability to solve the problem successfully. The third finding was that related rates of change problems that involved auxiliary problems were problematic for students and their ability to successfully solve the problems. The researcher recommends that calculus instruction should provide more opportunities for students to make sense of, and to solve non-routine related rates of change problems that have several quantities.

In a study focusing on student difficulties with related rates of change problems, Engelke (2004) researched to gain a better understanding of the obstacles that calculus students must overcome to have a conceptual understanding of related rates problem. Engelke asserted that students will not have a conceptual understanding of related rates of change without sufficient guidance. According to the study, textbooks usually give students 6-7 steps to follow when solving related rates of change problems. The most difficult related rates of change problems are those that require a student to solve an auxiliary problem first before solving a main equation. Students' difficulty with these types of problems are usually attributed to difficulties with both procedural and conceptual understandings. Students have deficiencies in concepts of variable, rate of change, functions (specifically composition), and derivative. According to Engelke (2004), to successfully solve related rates of change problems students should be proficient with their understanding of variable, functions, composition of functions, geometric properties, chain rule, and implicit differentiation. The major obstacles that many students face with related rates problems are: the inability to draw a picture that correctly represents the situation in the problem; not knowing what geometric relation is appropriate; not understanding implicit differentiation, the chain rule; manipulating symbols that have no meaning; and having a process-focused view of mathematics (Engelke, 2004).

There were three major difficulties that emerged from the data. Those difficulties were: algebraic and/or geometric deficiencies; student fixation on the procedural steps; and failure to recognize and consider general relationships. Engelke theorized that the lack of transformational/covariational reasoning is the root of the problem. The researcher found that students did not engage in transformational reasoning, creating mental model that can be manipulated to understand relationships, and did not actively engage in covariational reasoning, ability to coordinate change in a variable with change in another variable, at the beginning of the problems (Engelke, 2004; Carlson, et al., 2002). Student reliance on procedural steps, leads to random use of algebraic techniques and misguided geometric associations. Students not using transformational/covariational reasoning in solving problems results in students not being able to solve the required auxiliary problem. The tasks, in the study that were created by Engelke, were an attempt to help students to become proficient in creating an appropriate relationship by focusing on the generalities of the diagram and it led to very little progress. The researcher believes that students' inability to build a conceptual model of the situation, to identify the relevant relationships and to appropriately coordinate changes in these objects in their mind and also not applying transformational/covariational reasoning as the reasons students have difficulty in solving related rates of change problems (Carlson, et al., 2002; Engelke, 2004). Those reasons led to students engaging in procedural steps and not focusing on the relationships between quantities that are changing in the problem. Engelke suggests that students' transformational reasoning skills and how they apply them, especially in the diagramming phase are critical to successfully solving related rates of change problems.

In another study about conceptual and procedural knowledge of related rates of change, Martin (2000) examined university students' performance on geometric related rates of change problems. This study aimed to characterize students' ability to solve geometric related rates of change problems by identifying the conceptual and procedural knowledge required. The study also sought to identify which type of understanding is most closely related to successful performance.

One of the findings of the study was that the average score was below 60% on the geometric related rates of change problems. Martin attributed reasoning for the low scores to questions that incorporated auxiliary steps that require reasoning within the geometrical context of each problem. The auxiliary problems also required multiple steps to solve. Students had difficulty with both conceptual and procedural steps. Students were unable to translate previous success in geometry and algebra to success in calculus.

Similar to Mkhatshwa's (2020) and Engelke's (2004, 2008) studies, Martin (2000) concluded that calculus students are poor at solving geometric related rates problems especially those that require an auxiliary problem to be solved. The researcher suggested to proponents of the 'back to traditional' instruction that it too led to poor performance on steps linked to procedural knowledge as well as on steps primarily relying on conceptual understanding. It is critical for students to be confident, competent users of symbolic representations, and to make connections among verbal, symbolic, and graphical representations (Martin, 2000; Engelke, 2004). Both conceptual and procedural domains and the links between them receive considerably more attention throughout the school curriculum (Martin, 2000; Engelke, 2004).

## 2.6 Transfer

The concept of transfer has been studied has been studied extensively in the field of education. Transfer of learning is a crucial part of education. Transfer is the ability to apply previously learned knowledge or skills to new situations or problems. To succeed in both academic and non-academic settings, transfer is an essential skill that is a crucial component of learning. The traditional approach to teaching and learning has been to emphasize rote memorization and repetition. Recent research has emphasized the importance of actively noticing the underlying patterns and connections in the material being learned. Transfer is important to this study because of the similarity between the problems in Phase 1 and Phase 2 and I believe that participants will transfer the knowledge gained in one problem and use that knowledge to solve the similar problem.

Lobato and Siebert (2002) conducted a teaching experiment that investigated students' quantitative reasoning in a transfer situation. The researchers found four relationships between quantitative reasoning and transfer. The first relationship is that a students' procedural knowledge is not relevant to them when solving a quantitatively complex problem. The second relationship is that cognitive reconstruction of the quantitative relationships in a situation and the social support for a reconstruction of information play a critical role in quantitative reasoning and transfer. The third relationship is that students have the most difficulty solving problems that involve geometric similarity because of the necessity of having to think proportionally. The final relationship between quantitative reasoning and transfer is that students will benefit by having instruction that helps them develop quantitative reasoning.

Continuing their research in transfer, Lobato, et al. (2012) explored the role of noticing in transfer of learning processes. The researchers describe noticing as "selecting, interpreting, and working with particular mathematical features or regularities when multiple sources of information compete for students' attention" (Lobato et al., 2012, p. 438). The purpose of the study was to investigate whether noticing could be used to explain the occurrence of transfer in a classroom environment. The researchers believe that what students notice mathematically becomes central as a basis from which they generalize. The authors state that identifying centers of focus may not permit an accurate prediction of the fine-grained conceptions that will transfer, but it can inform general expectations regarding the nature of the transfer of learning. Lobato, et al. (2012) concluded that what students notice mathematically serves as the foundation that they use in a transfer situation.

Building on their previous study, Lobato, et al. (2013) further investigated the role of noticing in transfer of learning processes. The findings of the study were what students notice mathematically has consequences for their subsequent reasoning. The researchers conjectured that reasoning in subsequent situations comes from the generalizations of a student's learning experiences based on what was noticed mathematically. The researchers also noted that pinpointing centers of focus may not permit an accurate prediction of strategy use, but it can identify conjectures regarding the consequences of noticing for reasoning on novel tasks.

Lobato, et al.'s studies (2012, 2013) highlight the importance of noticing in transfer of learning processes. The studies suggest that noticing is a valuable tool for supporting transfer of learning processes and developing students' mathematical reasoning skills. The authors believe that these studies will help teachers become aware that for any mathematical topic there are multiple themes that their students may or may not notice and if students are not noticing the correct theme, it may be impossible for them to form the intended mathematical idea.

# Chapter 3

# Methodology

This sequential mixed methods study aims to explore students' use of problem-solving strategies when working paper-and-pencil related rates of change problems and how that utilization compares when working online related rates of change problems. It will also investigate how the use of online support systems affects students' use of problem-solving.

## 3.1 Setting

This study takes place at a large urban university in the Southwestern United States with an undergraduate enrollment of over 27,000. In the Spring of 2023, there were nine Calculus 1 sections. There were seven in-person sections and two sections that met exclusively online. Each in-person section had an enrollment cap of eighty students, while the online sections had an enrollment cap of seventy. All but two sections of the in-person classes were at capacity, one of the sections had an enrollment of seventy-nine students and the other had an enrollment of seventy-eight students. Due to the scheduling of the online courses and to ensure comparability of learning formats, the two online sections were not included in this study. Therefore, the seven in-person sections, with at total enrollment of 557 students, were used to draw data for this study.

All the first-semester calculus courses are coordinated by the Department of Mathematics and attend either two eighty-minute lectures or three fifty-minute lectures per week. Calculus 1 courses also had labs that met for fifty minutes twice a week. Each course section has two lab sections that meet twice a week for fifty minutes each session and have no more than 40 students each lab. In one lab session of the week students work in groups on problem-solving tasks, and the other lab session students have the opportunity to ask the instructor questions and are administered a quiz. In addition, the in-person sections had five distinct instructors since two instructors each were teaching two sections of Calculus I. Graduate teaching assistants facilitate the labs in conjunction with the professor of the course. Calculus 1 at this university has two midterm exams and one final exam in addition to homework and quizzes. Related rates of change is a topic covered around the mid-semester of the first-semester calculus course and it is assessed on the second midterm. Midterm 2 at this institution typically assesses learning of related rates of change with one free-response question that has multiple parts. At this university, students are given two hours to take midterms. The exam is typically structured as 22 problems, 20 of which are multiple choice and two free-response problems. The exams are departmental so, all calculus students take the same midterm exam in one of three similar versions.

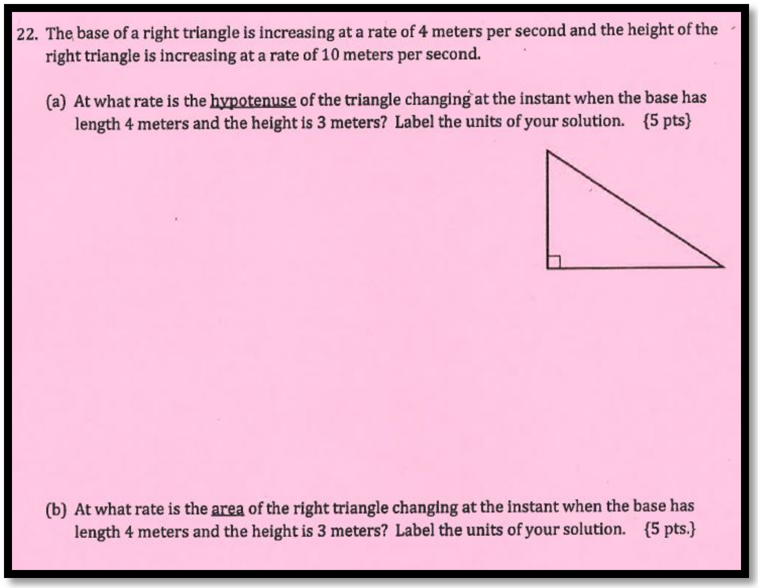
## 3.2 Procedures

I met with each of the seven Calculus 1 sections within the first two weeks of the Spring 2023 semester. Students were given an oral explanation of the study and a QR code that was linked to an overview of the study, a consent document, and a demographic survey for those that agreed to participate in the study (see Appendix A). Students gave consent for me to obtain online homework grades, online homework use of the ‘view an example’ feature, and Midterm 2 related rates of change question. Students completed the consent form and demographic survey outside of class and lab time. Students who completed the consent and survey were given a grade of 100 for a homework assignment. The students that chose not to participate were given an alternate assignment for that homework grade (see Appendix B). Consent was obtained from 317 students across all seven sections of the in-person Calculus 1 classes.

I observed two different instructors as they taught related rates of change to their classes to have a baseline understanding of the problem-solving strategies taught to the students. Instruction for related rates of change was provided for two lectures by each instructor. I also observed lab classes for both instructors when related rates of change was being covered. In the lab class, groups of 4-5 students worked collaboratively on five related rates of change problems. The lab assignment consisted of five related rates of change questions. The instructor gave a quick review of related rates of change at the beginning of the lab session. After the review, groups worked together on problems while the course and lab instructor actively moved around the classroom and answered questions. The lab assignment was submitted to the lab instructor and graded then it was returned back to the groups the next lab session.

*3.2.1 Midterm 2 Related Rates of Change (RRC) Scoring*

The related rates of change question that was given on Midterm 2 was a two-part question (see Figure 3-1). There were three different versions of the exam so the numbers in the problems were different in each exam. The question involved a right triangle that had an increasing base over a given time period. On the first part, students were asked to find the rate the hypotenuse was changing when the base and length of the triangle were a certain measurement. The second part of the question ask students to find the rate that the area is changing at those measurements.



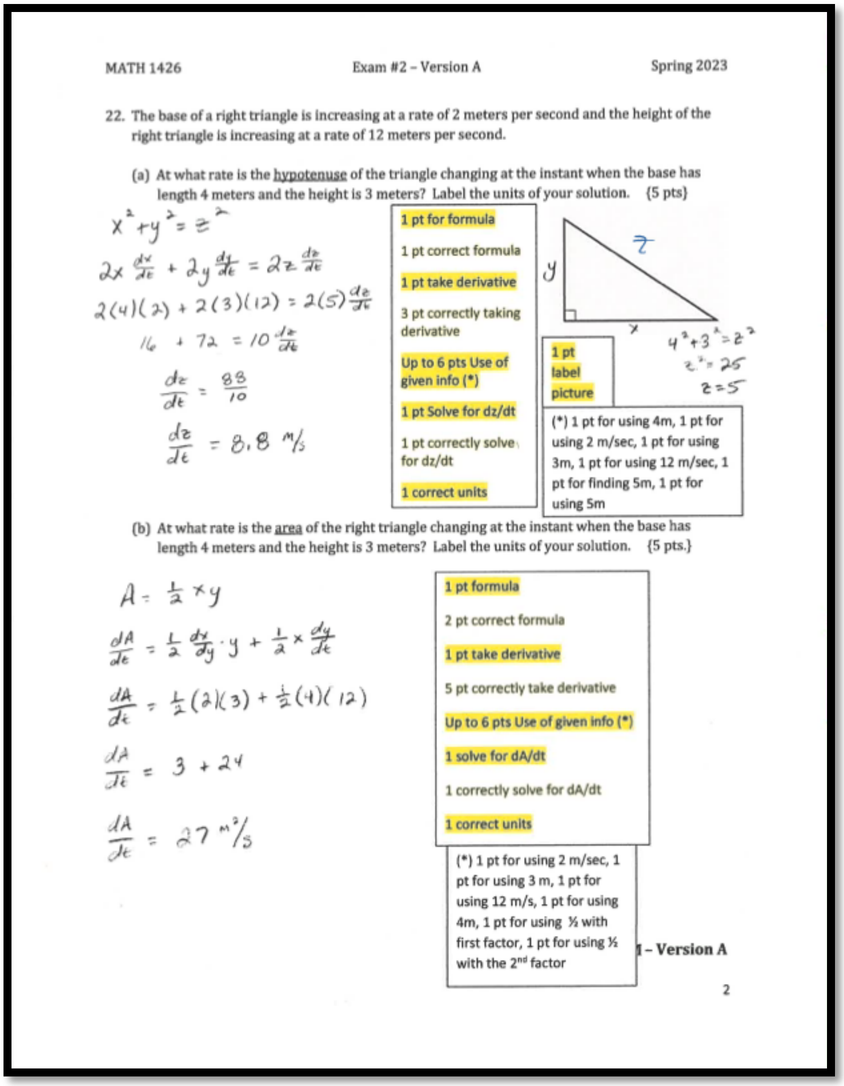
22. The base of a right triangle is increasing at a rate of 4 meters per second and the height of the right triangle is increasing at a rate of 10 meters per second.

(a) At what rate is the hypotenuse of the triangle changing at the instant when the base has length 4 meters and the height is 3 meters? Label the units of your solution. {5 pts}

(b) At what rate is the area of the right triangle changing at the instant when the base has length 4 meters and the height is 3 meters? Label the units of your solution. {5 pts.}

Figure 3-1 Midterm 2 (Version B) Related Rates of Change Problem

After Midterm 2 was completed, consenting students' related rates of change question was scanned and copied for grading. A rubric based on Engelke's (2008) "A Framework for the Solution for Related Rates Problem" was used to score the Midterm 2 RRC problems (See Appendix C). The rubric has a total of thirty-four points, 16 for part (a) and 18 for part (b). Of this 34-point total, 21 points were awarded for correct processes shown and 13 points were awarded for procedural work (see Figure 3-2).



MATH 1426 Exam #2 – Version A Spring 2023

22. The base of a right triangle is increasing at a rate of 2 meters per second and the height of the right triangle is increasing at a rate of 12 meters per second.

(a) At what rate is the <u>hypotenuse</u> of the triangle changing at the instant when the base has length 4 meters and the height is 3 meters? Label the units of your solution. {5 pts}

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(4)(2) + 2(3)(12) = 2(5)\frac{dz}{dt}$

$16 + 72 = 10\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{88}{10}$

$\frac{dz}{dt} = 8.8\ m/s$

1 pt for formula

1 pt correct formula

1 pt take derivative

3 pt correctly taking derivative

Up to 6 pts Use of given info (*)

1 pt Solve for dz/dt

1 pt correctly solve for dz/dt

1 correct units



$4^2 + 3^2 = z^2$

$z^2 = 25$

$z = 5$

1 pt label picture

(*) 1 pt for using 4m, 1 pt for using 2 m/sec, 1 pt for using 3m, 1 pt for using 12 m/sec, 1 pt for finding 5m, 1 pt for using 5m

(b) At what rate is the <u>area</u> of the right triangle changing at the instant when the base has length 4 meters and the height is 3 meters? Label the units of your solution. {5 pts.}

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dy}\cdot y + \frac{1}{2}x\frac{dy}{dt}$

$\frac{dA}{dt} = \frac{1}{2}(2)(3) + \frac{1}{2}(4)(12)$

$\frac{dA}{dt} = 3 + 24$

$\frac{dA}{dt} = 27\ m^2/s$

1 pt formula

2 pt correct formula

1 pt take derivative

5 pt correctly take derivative

Up to 6 pts Use of given info (*)

1 solve for dA/dt

1 correctly solve for dA/dt

1 correct units

(*) 1 pt for using 2 m/sec, 1 pt for using 3 m, 1 pt for using 12 m/s, 1 pt for using 4m, 1 pt for using ½ with first factor, 1 pt for using ½ with the 2nd factor

1 – Version A

2

Figure 3-2 Scoring Rubric with Process Point Criteria Highlighted

The scoring on part (a) corresponded to tracking the following criteria: labeling the given diagram; use of a formula, use of correct formula, taking the derivative, using the given information, solving the equation, correctly solving the equation, and answering with the correct units. For each of these criteria, zero points were awarded if no evidence was shown or one point otherwise with the exceptions of the criteria *correctly taking the derivative* which was assigned three points and of the criteria *use of given information* which was assigned up to six points. Part (b) points were awarded using the following criteria: use of a formula, taking the derivative, using the chain rule, solving the equation, correctly solving the equation, and answering with the correct units. Each of these criteria was awarded zero points if no evidence was shown or one point otherwise with the exceptions of the criteria *use of correct formula* which was awarded two points, the criteria *correctly taking the derivative* which was awarded five points since the chain rule is used, and the criteria *use of given information* which was awarded up to six points. Procedural points are based on procedural knowledge which is the ability to select and apply appropriate procedures required to solve a problem and to verify and justify the correctness of those procedures (Martin, 2000). Process points awarded corresponded to conceptual understanding of the process (Martin, 2000).

To recruit a diverse group of participants with respect to foundational knowledge and capacity for solving related rates of change problems, I examined statistical quartiles for the total points earned on the Midterm 2 related rates of change (RRC) problem and for the process point sub score earned on the Midterm 2 (RRC) problem. Midterm 2 RRC problem scores in the first quartile (i.e., bottom 25% of scores) for both total and process points were classified as *emerging*. Midterm 2 RRC problem scores in the second quartile for both total and process points were classified as *developing*. Midterm 2 RRC problem scores in the third quartile for both total and process points were classified as *proficient*. Midterm 2 RRC problem scores in the fourth quartile (i.e., top 25% of scores) for both total and process points were classified as *exemplary*. Midterm 2 RRC problem scores in different quartiles for total and process points were classified as *inconsistent*. Students with scores in this group were not considered for further participation in this study since I was aiming for a participant group whose process skills aligned with their procedural skills. The majority of the students, 82%, whose scores were categorized as *inconsistent* had a process point score in a higher quartile than their total points quartile. Also, students with Midterm 2 RRC problem scores in the first quartile were not considered for further participation in this study due to extremely limited use of problem-solving strategies on the assessment (see Figure 3-3). Further, students who were enrolled in Calculus 1 for the third time or more were excluded from participating in the task-based interviews. The reason for the exclusion of participants that have taken Calculus 1 more than two times is the belief that since these students have been taught related rates of change multiple times and their problem solving profiles could be quite different from those who are learning this for the first time or had only seen it once before. Students with Midterm 2 RRC problem scores classified as *exemplary*, *proficient*, and *developing* qualified to move on to the task-based interview for this study.

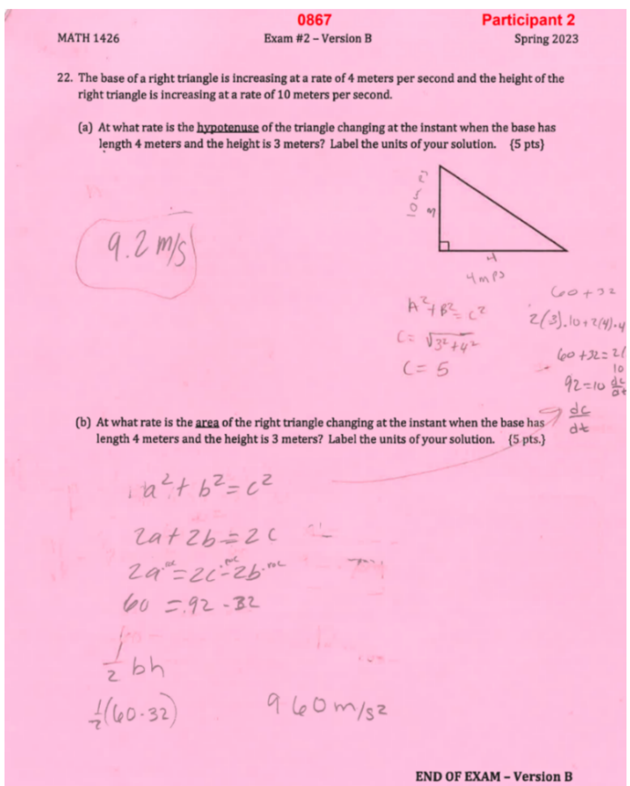



Figure 3-3 Example of Midterm 2 Student Work Categorized as *Emerging*

*3.2.2 Interview Invitation*

The interview invitations asked select students to participate in an up to sixty-minute task-based interview. The invitations also offered participants a gift card for participating in the interview session. Interview invitations were sent by email to an equal number of random students with scores in the *exemplary*, *proficient*, and *developing* categories, with the goal of having seven students from each category accept the invitation and participate in the task-based interview (see Appendix E). Interview invitation reminders were sent out if there was no response from the initial invitation. In the Spring 2023, a total of 145 invitations were sent to consenting participants (See Table 3-1). Fourteen students accepted and participated in the task-based interview. Those fourteen participants' Midterm 2 RRC problem scores fell into the following categories: six *exemplary*, six *proficient*, and two *developing* as defined in Section 3.2.1. The interview invitation explained the interview process and informed the student of the confidentiality of their interview should they wish to participate.

Table 3-1 Interview Invitations Sent vs. Accepted

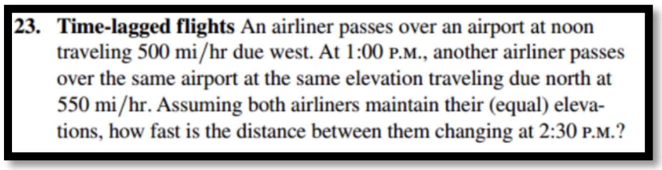
| Category | Invitations Sent | Invitations Accepted |
|---|---|---|
| ***Developing*** | 59 | 2 |
| ***Proficient*** | 38 | 6 |
| ***Exemplary*** | 48 | 6 |

*3.2.3 Interview Protocol*

Prior to the semester of implementation, I created and informally piloted the task-based interview protocol used in this study. The task-based interviews were limited to sixty minutes. The task-based interview required students to solve four related rates of change problems. Because I was interested in comparing problem-solving strategies when working paper-and-pencil versus online homework problems, two problems were chosen from the course's online homework platform, and the two related problems were chosen from the *Calculus Early Transcendentals* (Briggs et al., 2019) Calculus 1 textbook. Due to the structure of the online homework system, an online homework problem may give structurally similar problems to each student that only differ in the specific numbers or other data relevant in the problem. For example, on the online airliner problem, the times of when the airliners passed the airport and the time used for the final answer varied. On the online inverted water tank problem, the lengths and the water height varied but the radius was always one-half of the height regardless of the measurements given. The variations in the problems did not change the basis of the problems and they were fundamentally the same for all the participants.

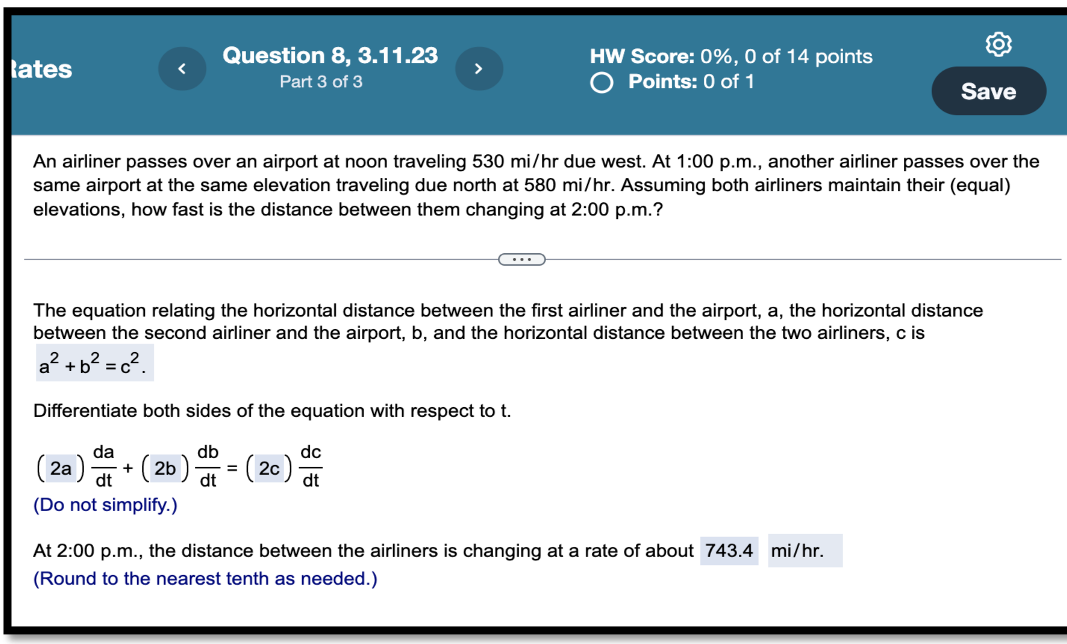
The interviews occurred in two phases. The first phase involved students solving a textbook paper-and-pencil problem (see Figure 3-4) followed by a similar online problem (see Figure 3-5). For reference, I call the first problem in this sequence Problem 1T to allude to its traditional paper-and-pencil format. Similarly, I call the second problem in this sequence Problem 2N to allude to its online format (using the second letter of "online"). During the first phase, students were asked to think aloud so that their problem-solving strategies were captured for analysis. I asked questions throughout the phase for clarification and to better understand the use of their problem-solving strategies (see Appendix F). For example, I asked, "Why did you relate the height and the radius?" Students were asked to solve each problem on paper their work was collected upon completion of each problem.

During the task-based interviews, participants were expected to work out their solutions on paper. Participants were expected to verbalize their thought process as they were working on the problems. They were also expected to explain the steps that they were taking to solve the problem and why they were taking those steps to solve the problem. If participants were not talking aloud, I would remind them to verbalize their thought process. I would also ask why they chose to make any step that was not verbalized. During the online problems, participants were asked to solve them as they would if they were at home working on homework.



**23. Time-lagged flights** An airliner passes over an airport at noon traveling 500 mi/hr due west. At 1:00 P.M., another airliner passes over the same airport at the same elevation traveling due north at 550 mi/hr. Assuming both airliners maintain their (equal) elevations, how fast is the distance between them changing at 2:30 P.M.?

Figure 3-4 Phase 1 Paper-and-Pencil Format Problem 1T



ates

Question 8, 3.11.23
Part 3 of 3

HW Score: 0%, 0 of 14 points
Points: 0 of 1

Save

An airliner passes over an airport at noon traveling 530 mi/hr due west. At 1:00 p.m., another airliner passes over the same airport at the same elevation traveling due north at 580 mi/hr. Assuming both airliners maintain their (equal) elevations, how fast is the distance between them changing at 2:00 p.m.?

The equation relating the horizontal distance between the first airliner and the airport, a, the horizontal distance between the second airliner and the airport, b, and the horizontal distance between the two airliners, c is $a^2 + b^2 = c^2$.

Differentiate both sides of the equation with respect to t.

$$(2a)\frac{da}{dt} + (2b)\frac{db}{dt} = (2c)\frac{dc}{dt}$$

(Do not simplify.)

At 2:00 p.m., the distance between the airliners is changing at a rate of about 743.4 mi/hr.
(Round to the nearest tenth as needed.)

Figure 3-5 Phase 1 Online Format Problem 2N

After solving each problem, participants were asked to walk me through their problem-solving process. They were also asked about the steps their professor taught them to solve a related -rates problems. Participants were asked about their confidence in solving similar problems. They were also questioned about their use or lack of use of the ‘view an example’ feature. Participants were also asked if the problem was similar to what they covered in class or lab.

After completing the first phase, participants began the second phase. In the second phase, students would solve an online homework problem (see Figure 3-6) followed by a similar textbook paper-and-pencil problem (see Figure 3-7). Since these were the third and fourth problems in the sequence of tasks, I use a similar naming scheme described for the first two problems. Similar to the procedures for the first phase, participants were then asked to think aloud so that their problem-solving strategies were captured for analysis. Questions were asked by the researcher throughout the phase for clarification and to better understand the use of their problem-solving strategies (see Appendix F). For example, participants were asked “Why did you relate the radius and the height to one another?”. Students were asked to solve each problem on paper that was collected at the completion of each problem.

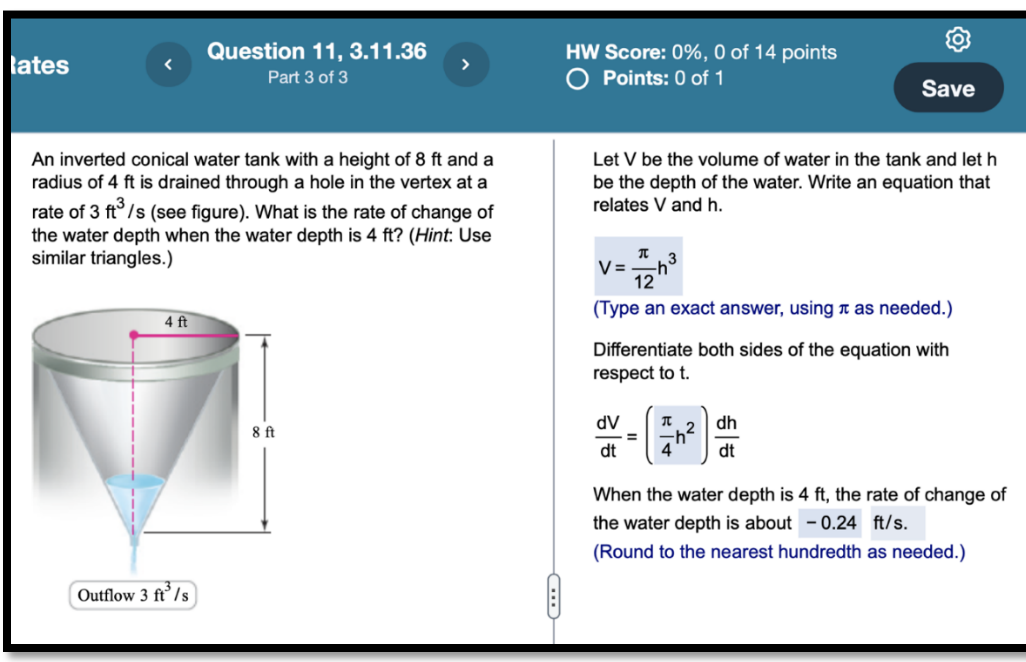



Figure 3-6 Phase 2 Online Problem 3N

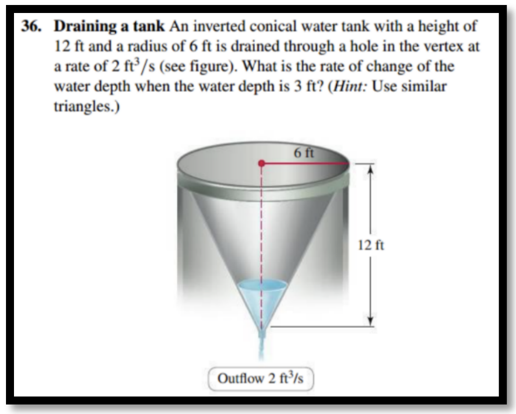



Figure 3-7 Phase 2 Paper-and-Pencil Problem 4T

The questioning during phase 2 was in a different order since the problems' modality were in different order than phase 1. Each task-based interview was recorded and transcribed. The recordings captured the participants' workspace and the computer screen when working on online problems.

## 3.3 Participants

There were a total of 14 students that agreed to participate in the task-based interview. The participants ranged from first-year freshmen to seniors. There was at least one participant from each professor that taught an in-person class. In order to preserve anonymity, participants were given pseudonyms based on their Midterm 2 score category and self-described gender identity. For example, a male whose Midterm 2 RRC problem score was classified as *proficient* could be given the name of *Paul* where the first letter of the name alludes to the "*proficient*" classification of his Midterm 2 RRC problem score.

### *3.3.1 Echo*

In the Spring 2023, Echo was a second-semester first-year freshman. This was her first time enrolling in Calculus 1. She reported that she had previously used online homework in a previous math course, and she does not use the 'view an example' feature very often. Echo also reported that she took Pre-Calculus in the Spring of 2022 and received an A in the course. Echo's Midterm 2 RRC problem scores were in the 4th quartile and were categorized as *exemplary*.

### *3.3.2 Eboy*

Eboy was also in his second semester of his first year of college. This was his 2nd time enrolling in Calculus 1. He had previous experience using online homework in a math course. Eboy also reported that he uses the 'view an example' feature very often. Eboy's Midterm 2 RRC problem scores were in the 4th quartile and were categorized as *exemplary*.

### *3.3.3 Earl*

Earl was also a freshman in his second semester of college. This was his first time enrolling in Calculus 1. He reported that he took Pre-Calculus the previous semester and earned a B in that class. Earl had experience with online homework and reported that he does not use the 'view an example' feature very often. Earl's Midterm 2 RRC problem scores were in the 4th quartile and were categorized as *exemplary*.

### *3.3.4 Ed*

Ed was in the second semester of his freshman year of college. He took Pre-Calculus in the previous semester and earned an A. Spring of 2023 was his first time enrolling in Calculus 1. Ed had previous experience with online homework in a mathematics course. Ed reported using the 'view an example' feature often when doing online homework. Ed's Midterm 2 RRC problem scores were in the 4th quartile and were categorized as *exemplary*.

### *3.3.5 Elsa*

Elsa was a freshman in her second semester of college. She did not take Pre-Calculus in her first semester and tested into Calculus 1. This was her first time taking

Calculus 1, and she had never used online homework for a mathematics course. Elsa's Midterm 2 RRC problem scores were in the 4th quartile and were categorized as *exemplary*.

*3.3.6 Eve*

Eve was in the second semester of her freshman year of college. She took Pre-Calculus the previous semester and earned a B in the course. This was her first time enrolling in Calculus 1. She reported that she has previous with online mathematics homework and that she uses the 'view an example' feature often. Eve's Midterm 2 RRC problem scores were in the fourth quartile and were categorized as *exemplary*.

*3.3.7 Paris*

Paris was a sophomore enrolled in Calculus 1 for the first time. She had previously taken College Algebra and Pre-Calculus at the university level. She had previous experience with online mathematics homework. Paris reported that she uses the 'view an example' feature very often. Paris's Midterm 2 RRC problem scores were in the 3rd quartile and were categorized as *proficient*.

*3.3.8 Pat*

Pat was a freshman in his second semester of college. He took Pre-Calculus the previous semester and earned a D. This was his first enrolling in Calculus 1. Pat had previous experience with inline mathematics homework, and he often uses the 'view an example' feature. Pat's Midterm 2 RRC problem score was in the third quartile and was categorized as *proficient*.

*3.3.9 Pamela*

Pamela was a second-semester freshman in her first year of college. She took Pre-Calculus the previous semester and earned a grade of A. She had previously taken College Algebra, Statistics, and Pre-Calculus at the university level. Pamela has previous experience using mathematics online homework. She did not report how often she uses the ‘view an example’ feature while completing online homework.

### *3.3.10 Penny*

Penny was a senior in the Spring of 2023. She had previously taken College Algebra, Trigonometry, and Pre-Calculus at the university level. This was her first time enrolling in Calculus 1. She has previous experience in online mathematics homework and uses the ‘view an example’ feature often. Penny’s Midterm 2 RRC problem score was in the 3rd quartile and was categorized as *proficient*.

### *3.3.11 Percy*

Percy was a sophomore in his second year of college. He had taken Pre-Calculus the previous semester and earned a B. He had also taken College Algebra his freshman year. He had previous experience using online homework in a mathematics course. Percy also reported that he uses the ‘view an example’ feature very often when doing online homework. Percy’s Midterm 2 RRC problem score was in the 3rd quartile and was classified as *proficient*.

### *3.3.12 Peter*

Peter was in his second year of college. He had previously taken College Algebra and Calculus 1 at the university level. This was his 2nd time enrolling in Calculus 1 and received a D the first time enrolling in the course. Peter has experience using online homework in mathematics courses and uses the ‘view an example’ feature very often. Peter’s Midterm 2 RRC problem score was in the 3rd quartile and was categorized as *proficient*.

### *3.3.13 David*

David was classified as a junior in the Spring of 2023. This was his second time enrolling in Calculus 1 course. He had no previous experience with online mathematics homework. David's Midterm 2 RRC problem score was in the 2nd quartile and was classified as *developing*.

### *3.3.14 Donald*

Donald was in the second semester of his freshman year. This was his second time enrolling in Calculus 1. He reported having previous experience with online mathematics homework and uses the 'view an example' feature very often. Donald's Midterm 2 RRC problem score was in the 3rd quartile and was classified as *developing*.

## 3.4 Data from Online Homework Platform Provider

I coordinated with the Mathematics Department's Assistant Department Chair to ask for her assistance with receiving any data on the related rates online homework from the online homework platform provider. The online homework textbook provider is also the textbook publisher and owner of the online homework system. I was put in contact with the Senior Director Data Insights and Analytics of the online homework provider. I requested data on the related rates of change section in the online homework system. The data I requested was specifically for the university that this study is taking place.

The data that I received from the Senior Director was the related rates of change online homework for the Spring of 2023. For the Spring 2023 semester all Calculus 1 students were assigned 14 problems on the related rates of change section. From the data there are six problems that more than 400 students answered. In five of those six problems over 50% of students used a help feature. A help feature is defined by the online homework platform provider as any of the following online features 'view an example', 'see a video', 'help me solve this', and 'ask my professor'. Although over 50% of students seeking help on those five problems, the overall success rate on the problems was fewer than 55% for four of those five problems.

## 3.5 Data Analysis

This mixed methods study used thematic analysis (Braun & Clark, 2006) to identify emergent codes that arose from the interviews. Deductive coding techniques were used and a priori codes derived from the research literature on mathematical problem solving and student learning of related rates were also used. The quantitative data generated from student work on midterms was used in the selection of participants for the interviews.

Qualitative coding methods and thematic analysis (Braun & Clark, 2006) were chosen to explore individuals' problem-solving strategies in-depth while trying to identify common themes among the cases. The audio-video recorded task-based interview, transcription, and individual participant's work were used to code each participant session based on mathematical problem-solving. A priori codes used for coding were based on a blend of Álvarez et al.'s (2019) characteristics or domains of problem-solving and Carlson and Bloom's (2005) Multidimensional Problem-Solving Framework (see Appendix G Codebook). Emergent codes were identified using open coding techniques (Braun & Clarke, 2006).

The a priori codes I used to code the transcripts are shown in Table 3-2. Coding of the transcripts also integrated observations from the recorded video and the written student work on the problems which was collected at the end of the interview session. Each instance in which a participant appeared to engage in a behavior listed in Table 3-2 was coded. An instance could be as short as a sentence statement or longer passages of a continuous thought or strategy. In addition, at the conclusion of each problem, participants were asked, “Walk me through your process of solving the problem.” After coding their transcripts, I used their description of their problem-solving process to triangulate the accuracy of my coding of the transcripts which corresponded to their real-time engagement in solving the RRC problems.

Table 3-2 Problem-Solving Codebook

| **Problem-Solving Coding** | **Behaviors** |
|---|---|
| Orienting/ Sense-making | The participant first makes sense of the problem by identifying key ideas and concepts. The participant may draw a picture, write the given information, and try to gain an understanding of the problem. |
| Planning | The participant accesses conceptual knowledge and heuristics as a means of constructing, imagining, and evaluating their conjectures. The participant verbalizes their strategy to approach the problem. |
| Representing/connecting | The participant reformulates the problem by using representation not already used in the problem or connects the problem to seemingly disjoint prior knowledge. The participant uses formulas or concepts not given in the original problem to solve the problem. |
| Executing | The participant accesses their conceptual knowledge, facts, and algorithms when constructing statements and carrying out computations. The participant attempts to solve the problem using higher-order techniques. |
| Reviewing: | The participant uses self-monitoring or self-assessing progress as problem- solving occurs or assessing the problem solution (e.g., checking for reasonableness) once the problem-solving process has concluded. The participant reviews work and makes corrections or assures themselves that they are doing the problem correctly. |
| Justifying | The participant communicates reasons for the methods and techniques used to arrive at a solution. The participant verbalizes their methods and the reasons why they are taking those steps |
| Checking: | The participant draws on their conceptual and procedural knowledge to verify the reasonableness of their results and the correctness of their computations. The participant verifies work, checks units, and reasonableness of their solution. |

Using open coding techniques three emergent patterns arose when analyzing the use of the ‘view an example’ feature (see Table 3-3**Error! Reference source not**

**found.**). These codes corresponded to the different uses of the 'view an example' feature arising in patterns of the data. Participants used the 'view an example' feature in three ways: to mimic, sense-make, or learn the process. Using 'view an example' by mimicking, the participant substituted their numbers into the similar example given by the program without considering other aspects of the problem. If a participant used 'view an example' to mimic the solution, problem-solving stopped at that point because they were no longer using problem-solving strategies and were just copying the solution. Using 'view an example' to learn a process manifested itself as instances where the participant tried to gain an understanding of how to correctly solve their problem by using the feature. Finally, when using 'view an example' as sense-making, a participant typically used the feature to help them understand an incorrect solution. Other participants indicated that they did not use 'view an example', so this was also in important "use" to note. Coding of the task-based interviews was done using Lumivero's NVivo qualitative data analysis computer software package.

Table 3-3 Emergent Uses of the 'View an Example' Feature

| Emergent Codes | Behavior |
|---|---|
| **View an example-mimic** | The participant follows what is shown in the example step-by-step and changes the numbers to theirs. |
| **View an example-process** | The participant looks at the example to understand the steps needed to solve the problem but does not use it as a template. |
| **View an example-sense-making** | The participant looks at the example problem to check if they were on the right track or to see why they were incorrect. |

The qualitative coding data from the task-based interviews were compared by modalities. I looked for trends based on these comparisons for implementation of problem-solving strategies, implementation of problem-solving strategies based on paper-and-pencil problem were given first or if online problem were given first, and implementation of problem-strategies when using ‘view an example’ when solving an online problem.

Data was gathered from the coding of the task-based interviews to see if and why participants accessed ‘view an example’ when solving the online problems. In addition, instances of “transfer” were coded that reflected instances when participants verbalized the use of transfer when solving Problems 2N and 4T of the task-based interviews where an episode was coded as transfer if a participant mentions that they are carrying over methods learned from a prior problem (but not simply mimicking).

## 3.6 Validity

Evaluating the trustworthiness of this investigation involves assessing the credibility, transferability, dependability, and confirmability (Stahl & King, 2020). For credibility, I draw upon my twenty years as an educator. During this time, I contributed to developing content to assess student learning. The assessments were evaluated for reliability and effectiveness. In this study, the coding process underwent rigorous self-review and periodic review by a more experienced researcher. I independently coded each task-based interview and compared the codes with a more experienced researcher. Any discrepancies within the coding were debated and resolved by the researchers.

To ensure transferability, I provided thorough details regarding the setting, the demographics, the participants, the interview protocol, and the coding process. Dependability and confirmability are attained through a meticulous description of the research procedures in the previous sections of this chapter as well as maintaining an audit trail to track the analysis process.

# Chapter 4

# Results

In this chapter, I report the scoring of the Midterm 2 RRC problem based upon the rubric described in Section 3.2.1 and also found in Appendix C. In addition, I report on the coding of the task-based interviews for the participants who participated in the interviews. I begin by presenting the Midterm 2 RRC problem scores and the categorization of the Midterm 2 RRC problem scores by quartile groups. Next, the self-reported demographic information for each of the participants from the task-based interviews along with a description of the problem-solving pathway I observed is discussed. This is followed by a presentation of the task-based interview transcript coding results. The task-based interviews are reported by problem and the results are given by the problem, by the participant, by the modality of the question, and by the achievement group. I also present the results of participant’s use of the help features that are offered in the online portion of the task-based interview. This is followed by the results from the data that indicate possible learning transfer when participants solved problems that were similar.

## 4.1 Midterm Exam

As described in Section 3.2.1, the free-response related rates of change Midterm 2 question was comprised of two parts (see Figure 3-1). Using the rubric developed for scoring the Midterm 2 RRC problems, participants could score 16 points on part (a) and 18 points on part (b) for a total of 34 points. Also as described in Section 3.2.1, 21 process points were used as a sub score of the 34-point total. Further, part (a) points were awarded by the following criteria: labeling the given diagram; use of a formula, use of correct formula, taking the derivative, using the given information, solving the equation, correctly solving the equation, and answering with the correct units. Each of these criteria was awarded zero points if no evidence was shown or one point if evidence. *Correctly taking the derivative* was awarded three points and the *use of given information* which was awarded up to six points. Table 4-1shows the point distribution for each criterion for part (a) of the midterm question.

Table 4-1 Midterm 2 RRC Problem Part (a) Scoring Criteria Presence

| **Part A (N=318)** | **Criteria Evident (frequency)** | **Criteria Not Evident (frequency)** |
|---|---|---|
| **Labeling the diagram** | 19 | 299 |
| **Use of a Formula** | 309 | 9 |
| **Use of Correct Formula** | 296 | 22 |
| **Take the derivative** | 276 | 42 |
| **Correctly Taking the Derivative** | 233 | 85 |
| **Solve the Equation** | 269 | 49 |
| **Correctly Solving the Equation** | 240 | 78 |
| **Answer with the Correct Units** | 276 | 42 |

The points for *use of given information* were awarded by how much of the given information from the problem statement was used to solve the problem. For example, if a participant used only two of the six pieces of the given information in the problem, they were awarded two points (Figure 4-1). For this problem, 288 participants earned all six points, 12 earned five points, 17 earned four points, 20 earned three points, 18 earned two points, two earned one point, and 21 earned zero points.

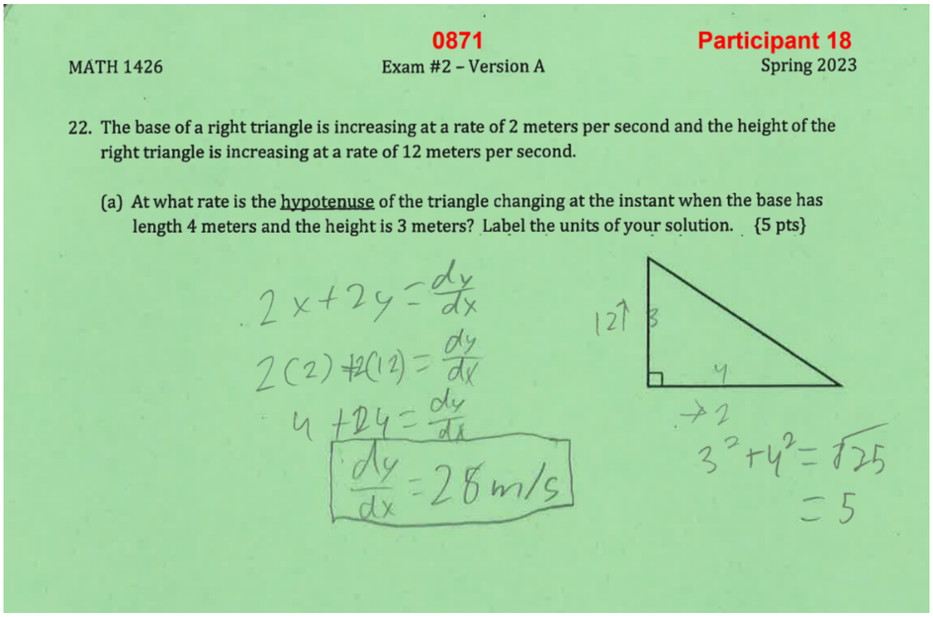



Figure 4-1 Sample Student Work Scoring Two Points for ‘Use of Given Information’

As shown in Table 4-1, 296 out of 318 participants used the correct equation (i.e., Pythagorean Theorem) for modeling the situation and 276 tried to compute the derivative, while only 233 then successfully computed the derivative. In addition, 78 participants incorrectly solved the equation that they chose to use for the problem.

For part (b) points were awarded by the following criteria: use of a formula, use of correct formula, taking the derivative, using the chain rule, solving the equation, correctly solving the equation, and answering with the correct units. Each of these criteria was awarded zero points if no evidence was shown or one point if evidence was shown. Correctly taking the derivative which was awarded five points since the chain rule is used, and using the given information which was awarded up to six points dependent on how many pieces of the given information were substituted into the equation. The data for part (b) is listed in Table 4-2.

Table 4-2 Midterm 2 Related Rates Part (b) Criteria Presence

| Part (b) (N = 318) | Criteria Evident (frequency) | Criteria Not Evident (frequency) |
|---|---|---|
| **Use of a Formula** | 287 | 31 |
| **Use of Correct Formula** | 255 | 63 |
| **Take the derivative** | 241 | 77 |
| **Correctly Taking the Derivative** | 89 | 229 |
| **Solve the Equation** | 242 | 76 |
| **Correctly Solving the Equation** | 236 | 82 |
| **Answer with the Correct Units** | 134 | 184 |

The data in Table 4-2 shows that of the 241 students computed a derivative, but only 89 students computed the derivative correctly to the correct equation to solve Part (b). As for the use of given information for Part (b), 128 participants used all the given information that was included in the problem and earned six points, one participant was earned five points, 21 was earned four points, 31 was earned three points, 88 was earned two points, one was earned one point, and 48 was earned zero points.

From this scoring, of 21 possible process points, participants had an average of 15.89. Participants also had an average of 23.52 for the total point score (out of 34). The median for the process points was 17 and the median for the total points was 25.

The midterm's process points data and total points data were used to create statistical quartiles for each category (see Table 4-3).

Table 4-3 Midterm 2 RRC Problem Quartiles For Process versus Total Points

| Quartiles | Process Points (21 possible) | Total Points (34 possible) |
|---|---|---|
| **Q1** | [0,14) | [0, 20) |
| **Q2** | [14, 17) | [20, 25) |
| **Q3** | [17, 20) | [25, 30.5) |
| **Q4** | [20, 21] | [30.5, 34] |

As described in Section 3.2.1, the data from the process points and total points quartiles were then used to group participants' Midterm 2 RRC problem scores into five categories *exemplary*, *proficient*, *developing*, *emerging*, and *inconsistent*.

Seventy-three Midterm 2 RRC problem scores were categorized as *inconsistent* because these scores corresponded to different quartiles for process points versus total points. Only 13 of the 73 *inconsistent* scores corresponded to a score in a higher quartile for total points versus process points, with 11 having a one quartile difference and two with a two quartile difference. In contrast, 60 out of the 73 Midterm 2 RRC problem scores that were categorized as *inconsistent* corresponded to a score in a higher quartile for the process score versus the total points score. For these 60 scores only one corresponded to a score having a two quartile difference and the other 59 had a one quartile difference for process versus total points scores. The number of participants with Midterm 2 RRC problem scores in each category is shown in Table 4-4.

Table 4-4 Participant Midterm 2 RRC Problem Score Categories by Quartile

| **Category** | **Number of Students (total = 318)** |
|---|---|
| ***Inconsistent*** | 73 |
| ***Emerging*** | 66 |
| ***Developing*** | 59 |
| ***Proficient*** | 46 |
| ***Exemplary*** | 74 |

## 4.2 The Participants

As described in Section 3.2.1, the participant Midterm 2 RRC problem score quartiles were used to determine the categories of groups of participants invited for interviews. The interview participants and their Midterm 2 RRC problem score categories are shown in Table 4-5. As mentioned in Section 3.3, the first letter of the participant's pseudonym corresponds to their Midterm 2 RRC problem score category.

Table 4-5 Interview Participants' Midterm 2 RRC problem score Categories

| ***Exemplary*** | ***Proficient*** | ***Developing*** |
|---|---|---|
| Echo | Paris | David |
| Eboy | Pamela | Donald |
| Earl | Pat | |
| Ed | Penny | |
| Elsa | Percy | |
| Eve | Peter | |

Each of the participants completed each problem. While not all of them were successful in arriving a correct solution, it is not the case that some problem-solving activity was truncated due to a student abandoning the problem or running out of time.

*4.2.1 Echo*

In the Spring 2023, Echo was a second-semester first-year freshman. This was her first time enrolling in Calculus 1. She reported that she had previously used online homework in a previous math course, and she does not use the 'view an example' feature very often. Echo also reported that she took Pre-Calculus in the Spring of 2022 and received an A in the course. Echo's midterm scores were in the 4$^{th}$ quartile and were categorized as *exemplary*.

On Problem 1T, Echo began by reading the problem. She drew and labeled a diagram of the airport, airplanes, and the airplane's speed. She then made the connection of the triangle in his diagram and Pythagorean Theorem. Echo then stated, "To find the rate of change, I need the derivative". She took the derivative, found the distance each airplane was from the airport, substituted in the values she had found, and solved the equation for $\frac{dc}{dt}$.

On Problem 2N, Echo read the problem and part 1 of the question. Part 1 was a drop down multiple-choice in which you had to choose the correct equation. Echo stated, “I tend to look at all the answers when I get the homework”. Echo also discussed how sometimes he does the homework in his head it depends if she wants to pull some paper out or not. Echo chose $a^2 + b^2 = c^2$ and received feedback from the online program that it was correct. Part 2 of the problem asked for the derivative of the equation. She derived the equation on the supplied paper. On part 3, she went to the drop-down box which was the units of the answer that had to be inputted and chose mph. Echo substituted the given info and the found distances into the derived equation and solved for the unknown. She entered his solution into the laptop and received instant feedback that her solution was correct.

Problem 3N began with Echo reading the problem and drawing and labeling her own diagram even though there was given in the problem. Echo did not remember the volume of a cone formula, so she googled it and wrote it down on his paper. The problem gave a hint to use similar triangles and Echo did this and related the radius and height. She then substituted the radius in terms of the height into the volume formula and entered the answer in part 1. Echo received feedback that his answer was correct. Part 2 of the problem asked for the derivative of the equation entered in part 1. Echo took the derivative, entered it into the laptop, and received feedback that it was correct. Echo substituted the given information into the derived equation and solved the equation. She entered the solution and received instant feedback that she was correct.

On problem 4T, Echo read the problem and wrote the formula for the volume of a cone down on paper. Echo then figured out the relationship of the radius and height with similar triangles. She then simplified the volume formula after inputting the radius in terms of the height. Echo took the derivative of the formula and explained why she was taking the derivative. She substituted in the given values and solved for the unknown.

During the task-based interview, Echo stated that all the problems were familiar and were similar to the ones covered in class or in lab. Echo also said that he does not use the ‘view an example’ often and only uses it if he doesn’t know the equation. She will watch a YouTube video if he doesn’t know how to do a problem because she feels ‘bad’ if she looks up the exact problem. Echo said that she was confident in her ability to solve related rates of change problems and she did answer all four problems correctly. When asked about the steps her professor taught her to solve related rates of change problems, Echo stated that to write out the problems and list all our variables and what we know and don't know.

*4.2.2 Eboy*

Eboy was also in his second semester of his first year of college. This was his second time enrolling in Calculus 1. He had previous experience using online homework in a math course. Eboy also reported that he uses the ‘view an example’ feature very often. Eboy’s midterm scores were in the $4^{th}$ quartile and were categorized as *exemplary*.

On Problem 1T of the task-based interview, Eboy read the problem and drew and labeled a diagram of the situation. He then recognized it was right triangle and stated that he would need to find the derivative of the Pythagorean Theorem. Eboy wrote down $2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$ then substituted the given speeds of the airplanes into $\frac{da}{dt}$ $and$ $\frac{db}{dt}$ of the equation and 1 into both $a$ $and$ $b$ and solved for $\frac{dc}{dt}$ . His justification for using 1 was that the problem stated that both planes were at the same elevation.

Eboy read Problem 2N and solved it on paper the same way he solved Problem 1T but this time he decided to use the times the planes departed the airport for $a$ $and$ $b$. He then solved the equation on the provided paper. I had to prompt Eboy to solve the problem in the online platform. Eboy chose the correct equation for part 1 and derived the equation for part 2 and received feedback from the online program that both answers were correct. He then entered his solution, 405 mph, for part 3 and received feedback that it was incorrect. After receiving the online feedback that his solution was incorrect, Eboy used the 'view an example' feature and mimicked the example problem.

On Problem 3N, Eboy read the problem, wrote down the equation and given information, and stated how he was going to solve the problem. Eboy then inputted the equation in part 1 without relating the radius and height. After seeing that his answer was incorrect, Eboy chose the 'view an example' feature and mimicked the example problem. He read and explained how the example problem was solved. Even after mimicking the example problem, Eboy still missed the last two attempts on part 3 of Problem 3N.

Eboy read Problem 4T and said "Oh, the same thing. You guys love clones." He then unsuccessfully tried to repeat the steps to the previous problem. Eboy incorrectly solved the equation for unknown variable.

Eboy said that he was confident in his ability to solve related rates of change problems since he did get them correct on his midterm. He stated that he uses the 'view an example very often and he did on both of the online problems in the task-based interview. Eboy finished Problem 1T in two minutes and thirty-seven seconds and Problem 4T in four minutes and twenty-four seconds. On Problem 2N, Echo spent 10 minutes and seven seconds solving. He spent 16 minutes and 38 seconds working on Problem 3N until he used all three attempts given on part 3. Eboy only solved Problem 2N correctly.

When asked about the steps that his professor taught him to solve related rates of change problems, Eboy stated that he was told to keep track of everything and write everything down and you will always have to "derive something."

*4.2.3 Earl*

Earl was also a freshman in his second semester of college. This was his first time enrolling in Calculus 1. He reported that he took Pre-Calculus the previous semester and earned a B in that class. Earl had experience with online homework and reported that he does not use the ‘view an example’ feature very often. Earl’s midterm scores were in the $4^{th}$ quartile and were categorized as *exemplary*.

Earl worked Problems 1T and 2N by reading the problem and drawing and labeling a diagram for both problems. He explained and justified the steps he was taking to solve the problems. He explained, justified, and reviewed more on Problem 1T than he did on Problem 2N.

Earl used the ‘view an example’ feature to get the equation of the volume of a cone. He was unable to enter the correct volume equation that related volume and height on the given three attempts. Earl the used the ‘view an example’ feature to see the correct of relating the two variables. Earl was able to differentiate the equation. On part 3, Earl was unable to correctly solve the problem in the three given attempts. Earl said that he misunderstood the question and was trying to find the change in the volume instead of the change in height. After incorrectly solving the problem, Earl reviewed the ‘view an example’ problem and worked the ‘help me solve it’ problem. The ‘help me solve it’ feature is similar to the ‘view an example’ except that you have to work out the problem and enter answers on all steps. On Problem 4T, Earl stated, “Since I did the last one, I now know what to do at least I hope so”. Earl was able to correctly solve Problem 4T. He drew a diagram, related the rates, justified, and reviewed his work to make sure that he was solving it correctly.

Earl stated during the task-based interview that he sparingly uses the 'view an example feature and only uses it when he gets stuck. Earl solved all four problems in the same manner. He did not seem to rely on the online features when solving the problems. When asked about the steps his professor taught him to solve related rates of change problems, Earl said drawing a picture and plugging in the information.

*4.2.4 Ed*

Ed was in the second semester of his freshman year of college. He took Pre-Calculus in the previous semester and earned an A. Spring of 2023 was his first time enrolling in Calculus 1. Ed had previous experience with online homework in a mathematics course. Ed reported using the 'view an example' feature often when doing online homework. Ed's midterm scores were in the $4^{th}$ quartile and were categorized as *exemplary*.

Ed solved Problems 1T and 2N using the same process. He did not verbalize any planning on Problem 2N as he did for Problem 1T. He did incorrectly solve both problems by using the same elapsed time for both planes to find the distance the airplanes were from the airport. Ed did not use the 'view an example' feature on 2N because he says that he only uses it after missing all attempts.

After reading Problem 3N and writing down the given information, Ed clicked on the 'view an example' feature and mimicked the example problem. On Problem 4T, Ed stated that it was the same problem. He wrote down the equation and the given information. Ed attempted to relate the rates as he had seen in the previous problem but was unable to correctly relate the radius and height. He did verbalize his confusion about relating the rates. Ed continued to work and complete the problem.

When asked about the steps his professor taught him to solve related rates of change problems, Ed stated, to find the function, take the derivative, and plug everything into the equation.

*4.2.5 Elsa*

Elsa was a freshman in her second semester of college. She did not take Pre-Calculus in her first semester and tested into Calculus 1. This was her first time taking Calculus 1, and she had never used online homework for a mathematics course. Elsa's midterm scores were in the 4$^{th}$ quartile and were categorized as *exemplary*.

On Problems 1T and 2N, Elsa basically used the same process to solve both problems. She made sure understood the problem, drew and labeled diagrams, connected the correct equation, and derived the equation. Elsa verbalized her plan and justified her steps. She did solve Problem 2N on paper before inputting any parts into the online program.

Elsa incorrectly answered her first two attempts on part 1of Problem 3N. She reread the problem and talked herself to get part 1 correct on the 3$^{rd}$ attempt. She correctly derived the equation after missing one attempt. On part 3, she missed her first attempt and received feedback to answer to the nearest hundredth. On the second attempt, she received feedback that the answer should be negative. Elsa did answer it correctly on her 3$^{rd}$ attempt. On Problem 4T, Elsa worked it out in the same manner as the previous problem. She verbalized planning and justifying more on this problem than she did on Problem 3N. Elsa explained that she doesn't use the 'view an example' feature and is able to find her own errors.

When asked about the steps she remembers her professor taught her to solve related rates of change problems, Elsa said to find the relationship between the variables to get the function, derive it, substitute, and solve the equation.

*4.2.6 Eve*

Eve was in the second semester of her freshman year of college. She took Pre-Calculus the previous semester and earned a B in the course. This was her first time enrolling in Calculus 1. She reported that she has previous with online mathematics homework and that she uses the ‘view an example’ feature often. Eve’s midterm scores were in the fourth quartile and were categorized as *exemplary*.

On Problem 1T, Eve began to quietly work on the problem and had to be reminded to think aloud. She worked out the problem and got an answer of 0.962 and said that it was incorrect because she was solving for the distance between the two planes. She then reworked the problem until she got an answer that seemed reasonable. On Problem 2N, she solved the first two parts correctly. On part 3, she chose the ‘view an example’ feature and mimicked the example problem. She said that she wanted to check if the way she was solving the problem correct.

Eve missed all three attempts of part 1 of Problem 3N. She had difficulty relating the rates. She also missed her first attempt of differentiating the volume equation. After missing her first attempt, Eve chose the ‘view an example’ feature and mimicked the example problem. On Problem 4T, Eve solved the problem with no difficulty. She was able to verbalize what she was doing and why she was taking those steps. After correctly solving Problem 4T, Eve stated that, “I used the same steps that was provided by the example”.

When asked about the steps her professor taught her to solve related rates of change problems, Eve said to write down the formula, plug it in, and to make diagrams.

*4.2.7 Paris*

Paris was a sophomore enrolled in Calculus 1 for the first time. She had previously taken College Algebra and Pre-Calculus at the university level. She had previous experience with online mathematics homework. Paris reported that she uses the 'view an example' feature very often. Paris's midterm scores were in the 3rd quartile and were categorized as *proficient*.

Paris answered Problem 1T with reading, drawing and labeling a picture. She used the Pythagorean Theorem and found the derivative. She substituted the given values into the formula but she made a mistake finding the distance the airplanes were from the airport. On Problem 2N, Paris was correct on parts 1 and 2. She chose the 'view an example' and mimicked the example problem but she was still unable to answer part 3 correctly.

After reading and writing down the given information for Problem 3N, Paris chose the 'view an example' feature and mimicked the results. Paris was able to solve Problem 4T correctly but she was unable to verbalize how she solved the problem. Paris said, "I don't understand why but I remember the example said I should".

When asked about the steps her professor taught her to solve related rates of change problems, Paris responded, to draw a picture, label it, plug in everything, figure out the formula, and take the derivative.

*4.2.8 Pat*

Pat was a freshman in his second semester of college. He took Pre-Calculus the previous semester and earned a D. This was his first enrolling in Calculus 1. Pat had previous experience with inline mathematics homework, and he often uses the 'view an example' feature. Pat's midterm score was in the third quartile and was categorized as *proficient*.

On Problem 1T of the task-based interview, Pat wrote out the given information and drew and labeled a diagram. He chose the Pythagorean Theorem as the equation to relate the rates. Pat found the distances of both airplanes at the time given in the problem, derived the equation, substituted the information into that equation, and solved the equation for the unknown value. On Problem 2N, Pat answered part 1 correctly on the first attempt and part 2 on the second attempt. After missing part 3 on the first two attempts, Pat gave up on solving the problem correctly.

On Problem 3N, Pat drew a diagram and labeled it with the given information. He then used the volume formula and attempted to take the derivative before giving up and using the 'view an example' feature and mimicked the example. Pat attempted to solve Problem 4T "just like the other problem". Pat attempted to solve the problem and became confused about solving the auxiliary problem. Pat was unable to move forward with the problem and gave up on trying to solve it.

When asked about the steps his professor taught him to solve related rates of change problems Pat stated, to draw a picture or diagram so that you can visualize it, write all of the given information, and that was basically it.

*4.2.9 Pamela*

Pamela was a second-semester freshman in her first year of college. She took Pre-Calculus the previous semester and earned a grade of A. She had previously taken College Algebra, Statistics, and Pre-Calculus at the university level. Pamela has previous experience using mathematics online homework. She did not report how often she uses the 'view an example' feature while completing online homework.

Pamela read, drew, and labeled a diagram, and used the Pythagorean Theorem for Problem 1T. She derived the formula and substituted the values into the wrong variables. She solved for her unknown and had a negative answer. Pamela was okay with having a negative answer even though it was unreasonable since it was the distance. On Problem 2N, Pamela went directly to the ‘view an example’ feature and mimicked the example problem. Pamela stated that she uses the ‘view an example’ because “she is scared of word problems”.

Pamela wrote down the given information for Problem 3N and before attempting to answer part 1 she chose the 'view an example’ and mimicked the results. On Problem 4T, Pamela was able to solve it correctly since “I just got to see an example”.

When asked about the steps her professor taught her to solve related rates of change problems she stated, to evaluate the rate, draw a picture, write everything down, write the variables in terms of the other, find the equation, take the derivative, and solve the function.

*4.2.10 Penny*

Penny was a senior in the Spring of 2023. She had previously taken College Algebra, Trigonometry, and Pre-Calculus at the university level. This was her 1st time enrolling in Calculus 1. She has previous experience in online mathematics homework and uses the ‘view an example’ feature often. Penny’s midterm score was in the 3rd quartile and was categorized as *proficient*.

Penny was quiet during the task-based interview and had to prompted multiple times to think-aloud. Penny was able to explain how and why she began to solve Problem 1T. Penny made a plan to solve the problem and gave reasons about her approach to the problem. She had trouble when it became time to substitute the values into the derived formula. Penny’s solution for the distance between the two planes was nine times the correct answer. On Problem 2N, Penny was correct on the first attempts for parts 1 and 2. Penny had difficulty solving part 3 and would start the problem over when she received the feedback that her answer was incorrect from the online platform.

Penny had trouble solving Problem 3N. She was unable to relate the volume and height to answer part 1. She attempted to use all of the given information in the formula leading to her having too many unknowns to solve the problem. Penny attempted to solve Problem 4T with the same method that she solved Problem 3N. Penny stated that she does not use the 'view an example' until she has missed the problem.

When asked about the steps her professor taught her to solve related rates of change problems she stated, to draw it out, figure out the formula, take the derivative, and solve it.

*4.2.11 Percy*

Percy was a sophomore in his second year of college. He had taken Pre-Calculus the previous semester and earned a B. He had also taken College Algebra his freshman year. He had previous experience using online homework in a mathematics course. Percy also reported that he uses the 'view an example' feature very often when doing online homework. Percy's midterm score was in the 3$^{rd}$ quartile and was classified as *proficient*.

Percy had trouble on how to solve Problem 1T. He drew and labeled his diagram. Percy used the Pythagorean Theorem and attempted to solve the problem from that point. He did not find the derivative and just solved for the hypotenuse of his diagram. He stated that he knew his solution was wrong. Percy used the 'view an example' feature on Problems 2N and 3N and mimicked the example for both problems. Percy was able to correctly solve Problem 4T. He stated that he was just trying to remember as much as he could from the problem he just did.

When asked about the steps his professor taught him to solve related rates of change problems he stated,

> "The steps, like exactly I remember her like saying some stuff to do over the lectures, but right now currently it's been like a few months since I

> actually like did this type of problem, so I don't remember the stuff that she told me to do, so I have to go back and refresh right since I have a final over this, but I'm not really that worried about it because I see I not seen it's going to be on the final, but it's only going to be a couple questions. But I didn't realize I just need to go over it again and then I'll know the thought process again, because I feel like I have the right thought process, but I'm just missing a couple of steps right here."

*4.2.12 Peter*

Peter was in his second year of college. He had previously taken College Algebra and Calculus 1 at the university level. This was his second time enrolling in Calculus 1 and received a D the first time enrolling in the course. Peter has experience using online homework in mathematics courses and uses the 'view an example' feature very often. Peter's midterm score was in the $3^{rd}$ quartile and was categorized as *proficient*.

On Problem 1T, Peter went through all the problem-solving process but he did not verbalize any planning. Peter used the correct formula and derived it correctly but he used the times that the planes departed the airport as the positions in the derived function. Peter was unsure of his solution and reviewed his work several times.

Peter chose the 'view an example' feature at the start of Problems 2N and 3N and mimicked the example for both problems. On Problem 4T, Peter attempted to "do the same thing" as the previous problem but he had trouble differentiating the volume formula and substituted the given into that formula and solved it.

When asked about the steps his professor taught him to solve related rates of change problems he stated, "Basically she always told us to write down what is given to us. And then because normally with related rates, you normally have to like draw out what the problem is asking you. And basically while doing that, then you can start figuring out where everything starts to go."

*4.2.13 David*

David was classified as a junior in the Spring of 2023. This was his second time enrolling in Calculus 1 course. He had no previous experience with online mathematics homework. David's midterm score was in the second quartile and was classified as *developing*.

On Problem 1T of the task-based interview, David was able to diagram the situation, use the correct formula, and differentiate the formula. He did not check his work or justify his strategy. David solved Problem 2N on his paper before inputting any answers in the online system. He did not have to check his solutions since the online system gave him feedback after entering his answer for each part of the problem.

David was unable to correctly solve parts 1 and 3 of Problem 3N and used the 'view example' to see how to solve it correctly. He was able to differentiate the volume formula on the first attempt. David stated that he only uses the view an example after he has attempted to solve the problem.

When asked about the steps that his professor taught him to solve related rates of change problems he stated, draw a picture, write down what the problem gave us, and then figure geometry formula we can use to help us solve the problem. And then. Take the derivative and plug in the information.

*4.2.14 Donald*

Donald was in the second semester of his freshman year. This was his second time enrolling in Calculus 1. He reported having previous experience with online mathematics homework and uses the 'view an example' feature very often. Donald's midterm score was in the 3$^{rd}$ quartile and was classified as *developing*.

Donald began solving Problem 1T with drawing and labeling a diagram. He recognized that he had to use Pythagorean Theorem and that it had to be derive to relate the rates. Donald went through all problem-solving domains while solving Problem 1T. Donald used the times as his positions in the derived function to solve the problem. On Problem 2N, Donald went to the drop-down box of part 1 without reading the problem and chose the correct equation. He also answered part 2 without reading the problem. Donald then chose the ‘view an example’ feature and mimicked the example for part 3 and he still missed all three attempts.

On Problem 3N, Donald chose the ‘view an example’ feature and mimicked the example problem. Donald stated that Problem 4T is the same problem as Problem 3N, and he was going to basically do the same thing. Donald did attempt to solve it the same and justified and reviewed his work, but he did not relate the radius and height correctly.

When asked about the steps his professor taught him to solve related rates of change problems he stated, “I remember that we have to draw a picture and to write out like the givens. That's about all I remember.”

## 4.3 Task-Based Interviews

The data from the coding of the audio-video recorded task-based interviews are presented in this section. Coding for problem-solving was counted for each instance that a domain was exhibited during the task-based interviews. For example, if a participant engaged in *justifying* at three different times while solving problem 1T that accounted for three identifications of *justifying.* The data presented is disaggregated by phase, problem, problem-solving domain, participant, achievement group, and modality.

### *4.3.1 Problem 1T Problem-Solving Coding*

Problem 1T of the task-based interview, Figure 4-2, is a textbook related rates of change problem that the Pythagorean Theorem is used to solve the problem. Problem 1T can be described as an introductory related rates of change problem because it is similar to the problems that are used when students are introduced to the topic.

**23. Time-lagged flights** An airliner passes over an airport at noon traveling 500 mi/hr due west. At 1:00 P.M., another airliner passes over the same airport at the same elevation traveling due north at 550 mi/hr. Assuming both airliners maintain their (equal) elevations, how fast is the distance between them changing at 2:30 P.M.?

Figure 4-2 Problem 1T of Task-Based Interview

As described in Section 3.5, the participant interview data was coded using thematic analysis (Braun & Clarke 2006). The code frequencies for each participant on Problem 1T of the task-based interview are shown in Figure 4-3. The bar graph in Figure 4-3 shows, for each participant, the frequency of each coded problem-solving strategy identified when analyzing the task-based interview transcripts for problem 1T.

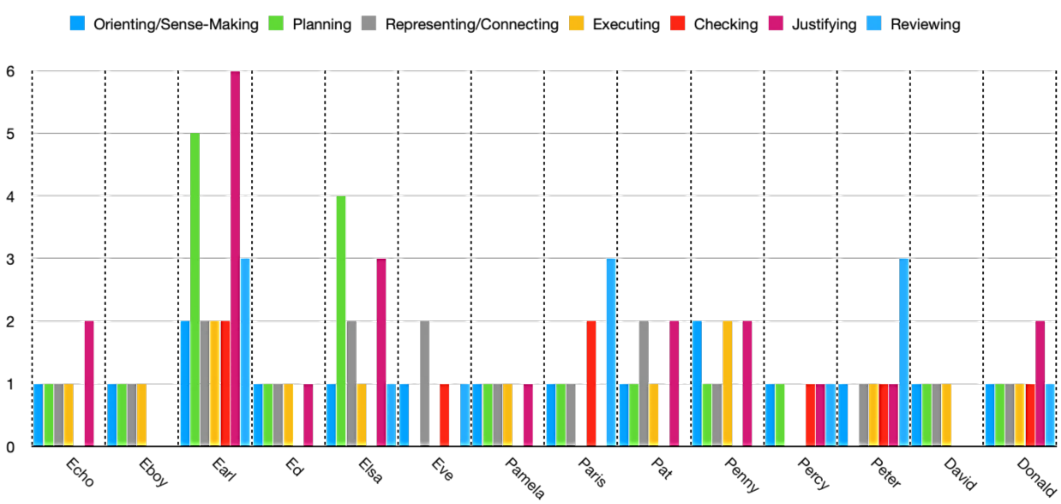



Figure 4-3 Problem 1T Problem-Solving Strategies Frequency by Student

On Problem 1T, the problem-solving code of justifying was identified most often when coding the task-based interviews. The problem-solving code of checking was identified least often for participants on Problem 1T. For Earl, I noted the highest number of problem-solving strategies, 22, compared to all other participants in the task-based interview.

Next, the frequency of each problem-solving domain for Problem 1T regardless of the participant's midterm score was tallied (see Table 4-6). Table 4-6 shows the frequency of the use of the problem-solving strategies for all participants on Problem 1T.

Table 4-6 Problem-Solving Frequency for all Participants Problem 1T

| Problem-Solving Dimension | Frequency |
|---|---|
| **Checking** | 8 |
| **Executing** | 13 |
| **Justifying** | 21 |
| **Orienting/Sense-Making** | 16 |
| **Planning** | 19 |
| **Representing/Connecting** | 17 |
| **Reviewing** | 13 |
| **Total** | **107** |

As seen in Table 4-6, justifying and planning were the top two problem-solving strategies identified when solving Problem 1 of the task-based interview. In contrast, only eight episodes of checking were identified.

To determine if problem-solving strategy identification varied widely among the participant midterm score groups the data was disaggregated to explore this (see Table 4-7). Since the *developing* group consisted of two participants and the *proficient* and *exemplary* groups had six participants each, the *expected number* was found to predict the frequency of each group if they all had the same number of participants.

Table 4-7 Problem-Solving by Midterm 2 RRC Problem Score Group Problem 1T

| | Developing | Proficient | Exemplary |
|---|---|---|---|
| **Checking** | 1 | 4 | 3 |
| **Executing** | 2 | 5 | 6 |
| **Justifying** | 2 | 7 | 12 |
| **Orienting/Sense-Making** | 2 | 7 | 7 |
| **Planning** | 2 | 5 | 12 |
| **Representing/Connecting** | 2 | 6 | 9 |
| **Reviewing** | 1 | 7 | 5 |
| **Frequency** | 12 | 41 | 54 |
| **Frequency per person** | 6 | 6.83 | 9 |
| **Expected number (based upon uniform distribution 107/14 per person)** | 15.29 | 45.86 | 45.86 |

As seen in Table 4-7, the problem-solving strategy use by the two participants in the d*eveloping* group accounted for only twelve instances overall while solving Problem 1T. The six participants in the p*roficient* group accounted for use of problem-solving strategies 41 times while solving Problem 1T of the task-based interview. The six participants in the *exemplary* group showed the greatest use of problem-solving strategies on Problem 1T with a frequency of 54. With a total number of 107 instances of problem-solving codes across all 14 participants, if we assume that this would be uniformly distributed across all participants the expected number of instances for each Midterm 2 RRC problem score group shows the *developing* group and the *proficient* group instances appear lower than expected. Examining the frequency per person in each Midterm 2 RRC problem score group shows a higher frequency per person for the exemplary group.

Table 4-8 Sample Excerpts for Problem 1T by Problem-Solving Strategy Code

| | |
|---|---|
| **Checking** | *Donald*: “I’m thinking I kind of messed up when it comes to the rate because the 500 miles per hour, that’s my...that’s the rate of change.”<br><br>*Paris*: “That doesn’t make sense. Okay, where did I go wrong?”<br><br>*Earl*: “Then to make sure you did everything correctly math-wise; you should get miles per hour since you’re getting the changing distance.” |
| **Executing** | *David*: “2C dc/dt equals 2A da/dt plus 2B db/dt. dc/dt equals. Divide the 2C on both sides so we get A da/dt plus db/dt. dt over c. So, we have A is 1250. times da/dt is 500 plus 825 times 550, db/dt over c, which is approximately 1497.707. 1250 times 500 plus 825 times 550. equals 1078750 over 1497.707. Within 720.268 equals Dcdt.”<br><br>*Eboy*: “And then derive that, and it will be 2a plus, or 2a by what's the derivative? And then 2b. 2C da/db” |
| **Justifying** | *Peter*: “The reason why I'm doing d over dt is because we're doing respect to time and t is time.”<br><br>*Elsa*: “And then now I'm just adding these together and then dividing by this number right here so I can isolate the DCDT, which is a variable I want to find out.” |
| **Orienting/ Sense-Making** | *Pamela*: “The first thing I would try to identify is I would use the variable, or not the variable, so dw over dt, because it's time. Yeah, so like the derivative of the west, I guess, the rate would be the 500.”<br><br>*Penny*: “First I'm drawing out what the problem says and then converting the amount [sic] of hours that have passed.” |
| **Planning** | *Ed*: “Then to find related rates, you do the derivative or to find, you want to find the distance between the changing.”<br><br>*Elsa*: “I'm just trying to think of how to find my other variables that I need to solve for the change of, or for the, the DCDT.” |
| **Repre-senting/ Connecting** | *Eboy*: “So, I guess it's a triangle, a right triangle. And then so I first have to get the derivative of the Pythagorean theorem. So, it's this a squared plus b squared equals c squared.”<br><br>*Donald*: “So, you would have to do Pythagorean theorem, which is a squared plus. b squared equals c squared.” |
| **Reviewing** | *Donald*: “I'm thinking I kind of messed up when it comes to the rate because the 500 miles per hour, that's my... That's the rate of change a little bit. Or taking it with the speed it's going at. So that has to be like a dA, dT, or a dB. DT, so. In this case, I'm looking for that. dc dt because that's the unknown so I'd have to do because it's given me the times so at noon and one…”<br><br>*Earl*: “Then to make sure that you did everything correctly math-wise, you should get miles an hour since you're getting the changing distance. Then you should get something along those lines.” |

*4.3.2  Problem 2N Problem-Solving Coding*

The online homework system scaffolded Problem 2N into three steps. The first step participants had to find the correct equation to solve the problem. On the second step participants were asked to differentiate the equation from step 1. Participants were asked to input the answer for the overall problem. Participants were given three attempts to answer each part of the problem before they are given the correct answer. Participants are also given hints anytime that they input an incorrect answer. Problem 2N (see Figure 4-4) of the task-based interview is the online problem corresponding to the textbook problem used in Problem 1T.

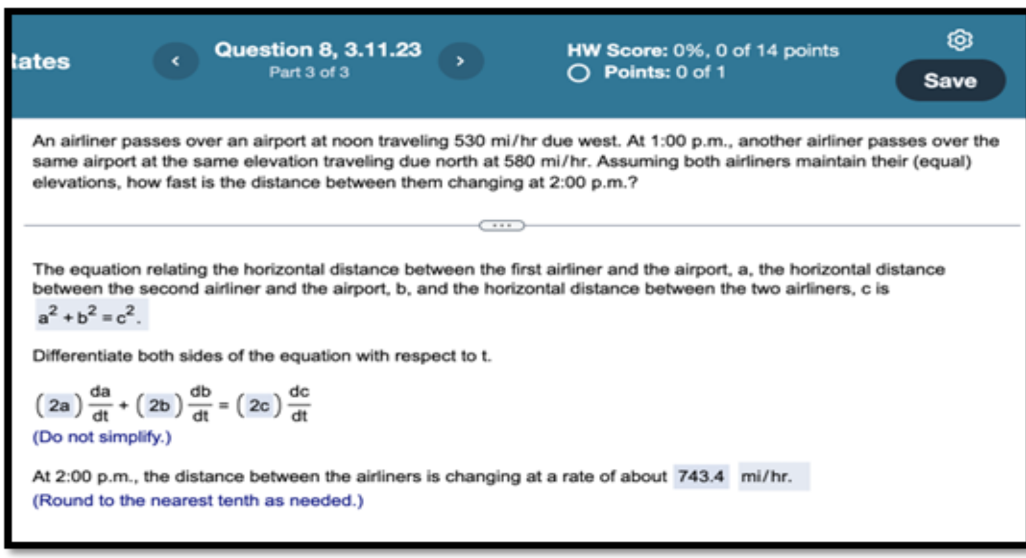



Figure 4-4 Phase 1, Problem 2N of the Task-Based Interview

The code frequencies identified for each participant on Problem 2N of the task-based interview are shown in Figure 4-5. In particular, the bar graph in Figure 4-5 shows, for each participant, the frequency of each coded problem-solving strategy identified when analyzing the task-based interview transcripts for Problem 2N.

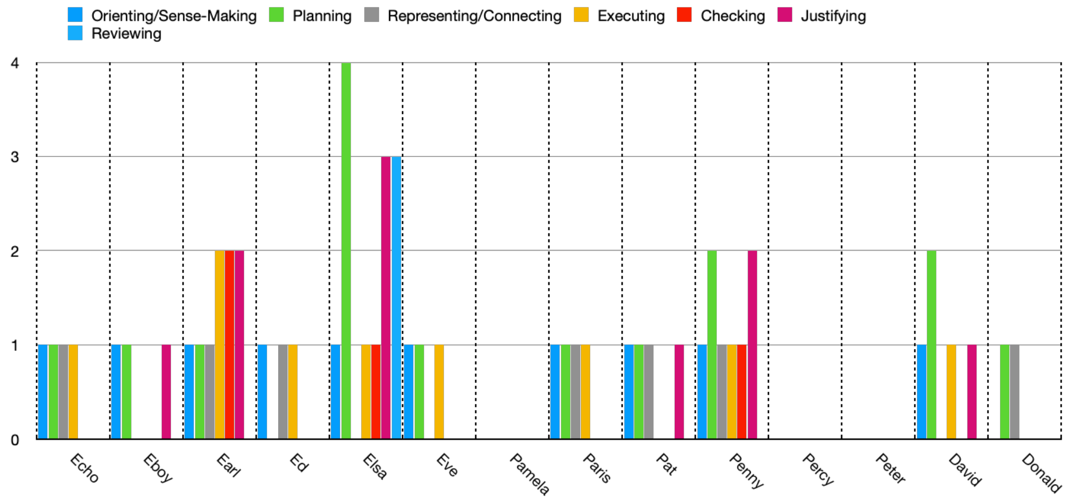



Figure 4-5 Problem 2N Problem-Solving Strategies Frequency by Student

For Pamela, Percy, and Peter no problem-solving strategies were identified in their solving process for Problem 2N (see Figure 4-5). Since Problem 2N was an online problem, participants could use the ‘view an example’ feature (more details will be given in Section 4.3), which these participants used to mimic the example problem. It is also shown that only Elsa used the problem-solving strategy of reviewing.

Next, the frequency of each problem-solving domain for Problem 2N regardless of the participant’s midterm score was tallied (see Table 4-9). Table 4-9 shows the frequency of the use of the problem-solving strategies by all achievement level categories on Problem 2N.

Table 4-9 Problem-Solving Use by all Participants Problem 2N

| **Problem-Solving Domain** | **Frequency** |
|---|---|
| **Checking** | 4 |
| **Executing** | 9 |
| **Justifying** | 10 |
| **Orienting/Sense-Making** | 10 |
| **Planning** | 15 |
| **Representing/Connecting** | 7 |
| **Reviewing** | 3 |
| **Total** | **58** |

As seen in Table 4-9, the problem-solving strategy of planning was used with the most frequency while solving Problem 2N, in contrast, checking was the least utilized problem-solving strategy while solving the online Problem 2N.To determine if problem-solving strategy identification varies widely among the participant midterm score groups the data was disaggregated to explore this (see Table 4-10).

Table 4-10 Problem-Solving by Midterm 2 RRC Problem Score Group Problem 2N

| | **Developing** | **Proficient** | **Exemplary** |
|---|---|---|---|
| **Checking** | 0 | 1 | 3 |
| **Executing** | 1 | 2 | 6 |
| **Justifying** | 1 | 3 | 6 |
| **Orienting/Sense-Making** | 1 | 3 | 6 |
| **Planning** | 3 | 4 | 8 |
| **Representing/Connecting** | 1 | 3 | 3 |
| **Reviewing** | 0 | 0 | 3 |
| **Frequency** | 7 | 16 | 35 |
| **Frequency per person** | 3.5 | 2.67 | 5.83 |
| **Expected number (based upon uniform distribution 58/14 per person)** | 8.29 | 24.86 | 24.86 |

As seen in Table 4-10, the use of problem-solving strategies for those in the *exemplary* group was greatest followed by the *proficient* group and the *developing* group respectively. With a total number of 58 instances of problem-solving codes across all 14 participants, if we assume that this would be uniformly distributed across all participants the expected number of instances for each Midterm 2 RRC problem score group shows the *developing* group and the *proficient* group instances are lower than expected.

Table 4-11 shows some example excerpts of the problem-solving coding of the task-based interview transcripts from Problem 2N.

Table 4-11 Sample Excerpts by Problem-Solving Strategy Code (Problem 2N)

| | |
|---|---|
| Checking | *Elsa*: [checking her work] "And then that is the distance between the two airports at time two. And then this is the distance from the, or from, this is distance between the two airlines at time t. This is the distance between the airport and the first airline or second airline. And then this is the distance between. the airline and the airport, the second one in time t, and then I'm going to say the rate at which this is changing is... 520 miles per hour. And this is the rate at which the second is changing" |
| Executing | *Echo*: "And then you can solve for that. The hours just cross out. This is 580 miles per hour. And then here, the hours just cross out. 2 times 430. So now I will list those variables. So, A equals 1060, B equals 1801. So, C, just using this equation, 156 squared plus 580 squared."<br><br>*Penny*: "With the Pythagorean theorem, I'm going to take the derivative again with respect to t since it's change in time." |
| Justifying | *David*: "So, to find the rate of change. We have to do the derivative of the Pythagorean theorem."<br><br>*Penny*: "And now that I have my ABC and all my other variables, I can plug them into the derivative." |
| Orienting/Sense-Making | *Paris*: "Okay, I make pictures of everything. So, I draw a little compass as it has like directions. This is a little bit of an airport due East; he's coming that way. Like 580 to south."<br><br>*Pat*: "I'll draw a diagram. So, there's an airport airplane going west. And then, one going to the south. One passes over at noon. The other passes over at 1. So. at one o'clock. The distance that airplane B has traveled is the rate is going at. times one because it's been one hour. So that's the sense for that one. and then for airplane A. It's... 500, the rate is going up, times 2, because it's been 2 hours, so that's 1000. And at two o'clock, it would be..." |
| Planning | *David*: "So, to find the rate of change. We have to do the derivative of the Pythagorean theorem."<br><br>*Elsa*: "And then I'm going to set it up so that I can have a triangle in this problem so that I can do the Pythagorean theorem to find the rate at which the distance between them is changing." |
| Rep./Con. | *Donald*: "That it's got to be Pythagorean theorem so that's a squared plus b squared c squared."<br><br>*Paris*: "Oh, this is gonna be the Pythagorean stuff. okay I think the square, I think it was square and then. a squared b squared equals c squared." |
| Rev. | *Elsa*: "And then that is the distance between the two airports at time two. And then this is the distance from the, or from, this is distance between the two airlines at time t." |

*4.3.3 Comparing Problem-Solving on Phase 1 Problems*

As described in the Methodology, Phase 1 of the task-based interview consisted of two related rates of change problems. The two problems were almost identical and only differed by the numerical values presented in the problems. Problem 1T is a traditional textbook paper-and-pencil problem, and Problem 2N is an online problem. Table 4-12 shows the frequency of use of the problem-solving strategies for each participant for Phase 1 of the task-based interview.

Table 4-12 Phase 1 Problem-Solving Frequency per Problem

| Student | Problem 1T | Problem 2N |
|---|---|---|
| **Echo** | 6 | 4 |
| **Eboy** | 4 | 3 |
| **Earl** | 22 | 9 |
| **Ed** | 5 | 3 |
| **Elsa** | 12 | 13 |
| **Eve** | 5 | 3 |
| **Pamela** | 5 | 0 |
| **Paris** | 8 | 4 |
| **Pat** | 7 | 4 |
| **Penny** | 8 | 8 |
| **Percy** | 5 | 0 |
| **Peter** | 8 | 0 |
| **David** | 4 | 5 |
| **Donald** | 8 | 2 |

As seen in Table 4-12, all participants, except for Elsa, Penny, and David, used more problem-solving strategies while solving the traditional textbook paper-and-pencil problem (Problem 1T) than they used to solve the online problem (Problem 2N) during Phase 1 of the task-based interview. From Table 4-12, I also note that Earl's relatively heavy use of problem-solving may be skewing the problem-solving totals for is Midterm 2 RRC problem score group.

*4.3.4 Problem 3N Problem-Solving Coding*

The online homework system scaffolded Problem 3N into three steps. In the first step, participants had to find the correct equation to solve the problem. Participants were asked to differentiate the equation from step 1 on the second step. Participants were asked to input the answer for the overall problem for the third step of the problem. Participants are given three attempts to answer each part of the problem before they are given the correct answer. Participants are also given hints anytime that they input an incorrect answer. Problem 3N of the task-based interview, Figure 4-6, is an online related- rates of change homework question. Problem 3N may be viewed as a more challenging related rates of change problem since it requires solving an auxiliary problem before being able to use the cone formula to relate the variables.

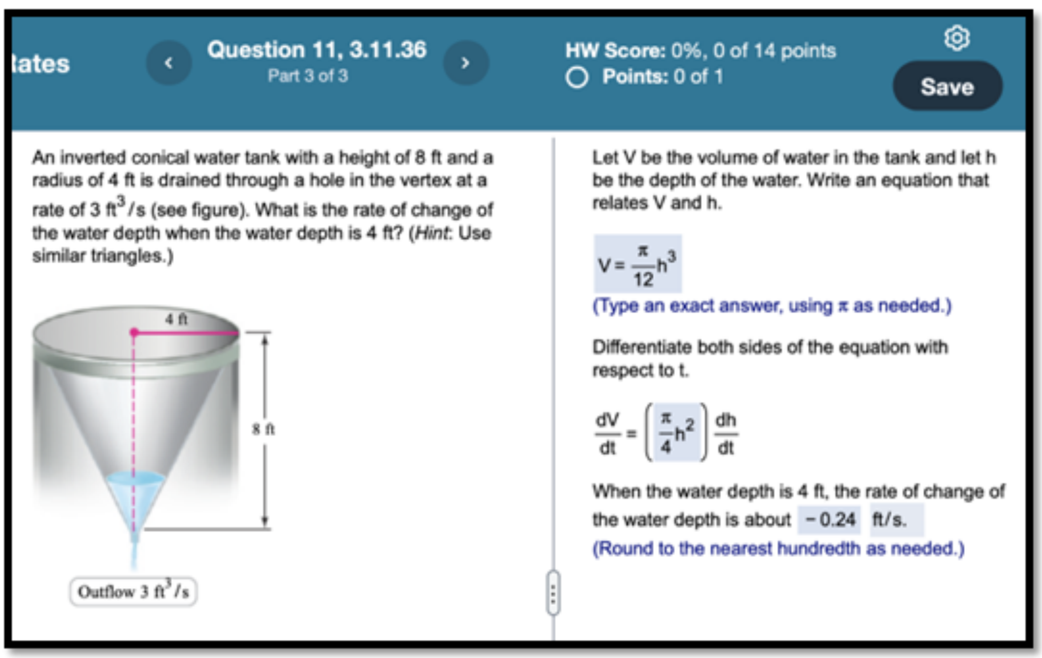



Figure 4-6 Phase 2 Online Homework Problem (3N)

The code frequencies for each participant on Problem 3N of the task-based interview are shown in Figure 4-7. In particular, the bar graph in Figure 4-7 shows, for each participant, the frequency of each coded problem-solving strategy identified when analyzing the task-based interview transcripts for Problem 3N.

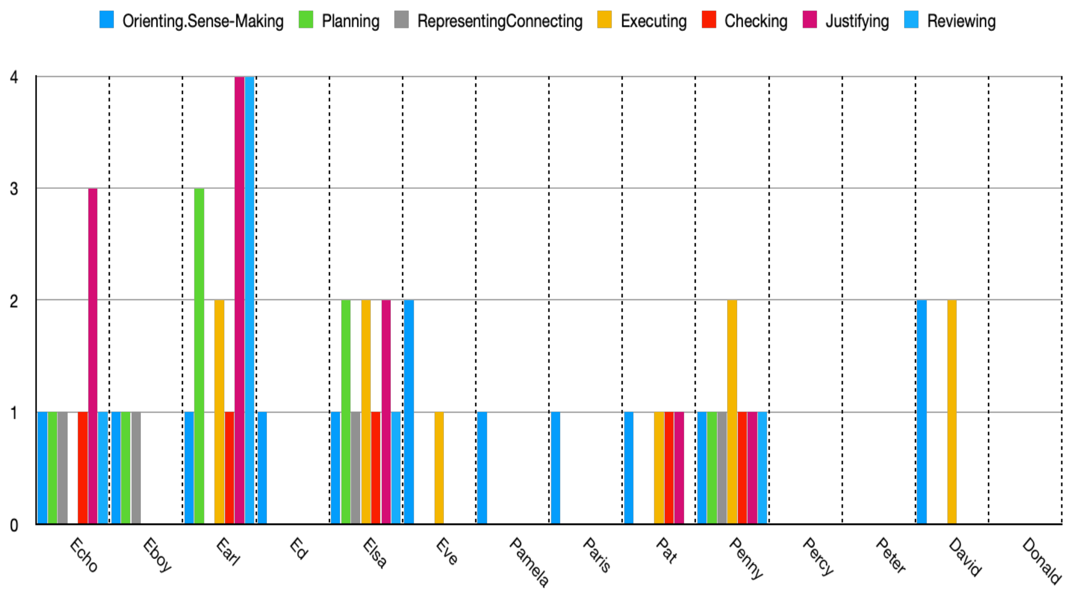



Figure 4-7 Problem 3N Problem-Solving Strategies Frequency by Student

As shown by Figure 4-7, for Percy, Peter, and Donald, no problem-solving strategies were identified in the problem-solving process. Since Problem 3N is also an online problem, participants could use the 'view an example' feature (more details will be given in Section 4.3), which these participants used to mimic the example problem. Echo, Elsa, and Penny did not use the 'view an example' feature and utilized more problem-solving strategies than those that did use the feature. Ed, Paris, and Pamela attempted to solve Problem 3N on their own and after missing some of the attempts they then used the 'view an example' feature.

Next, the frequency of each problem-solving domain for Problem 3N regardless of the participant's midterm score was tallied (see Table 4-13). Table 4-14 shows the frequency of the use of the problem-solving domain's frequency as used by all achievement level categories on Problem 3N.

Table 4-13 Problem 3N Problem-Solving Frequencies

| | **Frequency** |
|---|---|
| **Checking** | 5 |
| **Executing** | 10 |
| **Justifying** | 11 |
| **Orienting/Sense-Making** | 13 |
| **Planning** | 8 |
| **Representing/Connecting** | 4 |
| **Reviewing** | 7 |
| **Total** | **58** |

As seen in Table 4-13, the problem-solving strategy of orienting/sense-making was used with the most frequency while solving Problem 3N, in contrast, the least utilized problem-solving strategy while solving Problem 3N was representing/connecting.

To determine if problem-solving strategy identification varies widely among the participant midterm score groups the data was disaggregated to explore this (see Table 4-14).

Table 4-14 Problem-Solving by Midterm 2 RRC Problem Score Group Problem 3N

| | Developing | Proficient | Exemplary |
|---|---|---|---|
| **Checking** | 0 | 2 | 3 |
| **Executing** | 2 | 3 | 5 |
| **Justifying** | 0 | 2 | 9 |
| **Orienting/Sense-Making** | 2 | 4 | 7 |
| **Planning** | 0 | 1 | 7 |
| **Representing/Connecting** | 0 | 1 | 3 |
| **Reviewing** | 0 | 1 | 6 |
| **Frequency** | 4 | 14 | 40 |
| **Frequency per person** | 2 | 2.33 | 6.67 |
| **Expected number (based upon uniform distribution 58/14 per person)** | 8.28 | 24.86 | 24.86 |

As seen in Table 4-14, the *developing* group showed the least use of problem-solving strategies on Problem 3N by utilizing executing and orienting/sense-making on two occasions each. With a total number of 58 instances of problem-solving codes across all 14 participants, if we assume that this would be uniformly distributed across all participants the expected number of instances for each Midterm 2 RRC problem score group shows the *developing* group and the *proficient* group instances are much lower than expected.

Table 4-15 shows some examples of the problem-solving coding of the task-based interview transcripts of Problem 3N.

Table 4-15 Sample Excerpts by Problem-Solving Strategy Code (Problem 3N)

| | |
|---|---|
| Checking | *Earl*: "Oh, right. Since it's decreasing, I forgot about that. Since it's decreasing, it's negative."<br><br>*Pat*: "This seems wrong. The fraction is really weird. So, it is like, I don't know, this is a big denominator." |
| Executing | *David*: "All right, differentiate both sides of the equation with respect to t."<br><br>*Earl*: "Can you differentiate both sides in respect to t? Should be. Since these two are basically constants, we can pull those out. Using power rule for this one, you get 3h squared dh dt. Like chain rule. I guess I could reduce that…" |
| Justifying | *Echo*: "I even read the question. I was confused because I was like, why do I have this equation, but I don't need to plug anything into it because I don't need to solve for volume because the derivative of V is just dV dt. But I just needed to find an equation that relates V and H…"<br><br>*Penny*: "Oh, that's another way I know its volume is because the rate of change is feet cubed. And cubed usually goes with volume." |
| Orienting/Sense-Making | *Earl*: "So, I start by writing everything that I know. What's the rate of change from the water depth is? What's dz/dT when H is 5 feet? OK. I guess I could do D-O-D-T. I tried to make the variables be as different from each other as possible, because my handwriting isn't very good. So, it can be easy to get confused for me. But yeah. Like it says there, it says use similar triangles. And that's what a professor taught us. But again, not very good with triangles."<br><br>*Elsa*: "Okay so I'm just going to write out the variables that I have. So, I have the height. The radius. And then. Then I have the rate at which the height is changing, or the volume is changing…" |
| Planning | *Echo*: "I just have to write the equation right now. I even read the question. I was confused because I was like, why do I have this equation, but I don't need to plug anything into it because I don't need to solve for volume because the derivative of V is just dV dt. But I just needed to find an equation that relates V and H."<br><br>Echo: "Well, we should be taking the derivative, of course. We were going to do that at some point." |
| Rep/Con | *Eboy*: "it should be area of the circle, pi r squared times height, or 4 divided by 3 pi r squared. So, v equals such that expression that I'm using an expression. In terms of h" |
| Reviewing | Echo: "Oh, right. Since it's decreasing, I forgot about that. Since it's decreasing, it's negative."<br><br>*Earl*: "But if we take the derivative of this, we're taking the derivative of everything. And I know r and h, but I don't know dr and dh. Question that relates V and H. I guess first I should answer these. So, we can do 6 pi h squared." |

*4.3.5 Problem 4T Problem-Solving Coding*

Problem 4T, Figure 4-8,of the task-based interview is the corresponding textbook question that was used as our Problem 3N. Problem 4T can also be characterized as a more challenging related rates of change problem since it also requires solving an auxiliary problem before using the volume of a cone formula to relate the variables.

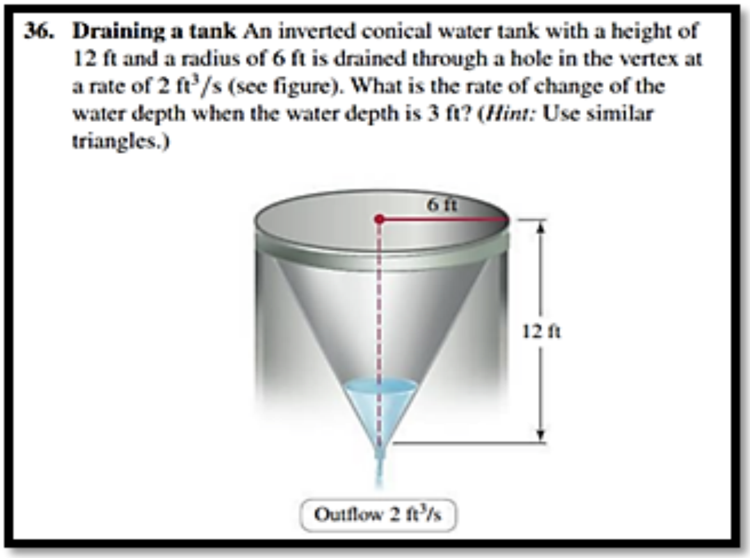

**36. Draining a tank** An inverted conical water tank with a height of 12 ft and a radius of 6 ft is drained through a hole in the vertex at a rate of 2 $ft^3/s$ (see figure). What is the rate of change of the water depth when the water depth is 3 ft? (*Hint:* Use similar triangles.)


Figure 4-8 Problem 4T of Task-Based Interview

The code frequencies for each participant on Problem 4T of the task-based interview are shown in Figure 4-9. In particular, the bar graph in Figure 4-9 shows, for each participant, the frequency of each coded problem-solving strategy identified when analyzing the task-based interview transcripts for Problem 4T.

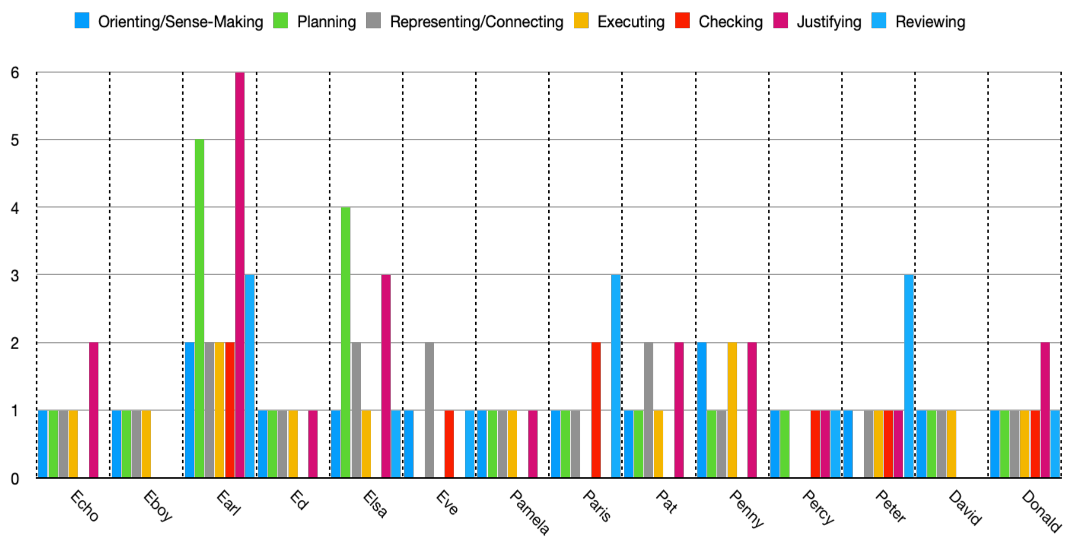



Figure 4-9 Problem 4T Problem-Solving Strategies Frequency by Student

As shown by Figure 4-9, shows that every participant used at least one problem-solving strategy while attempting to solve Problem 4T. Ebony and Peter were identified in their solving process for Problem 4T to have used only one problem-solving strategy (see Figure 4-9). It is also shown in Figure 4-9 that Earl used the most problem-solving strategies while solving Problem 4T.

Next, the frequency of each problem-solving domain for Problem 4T regardless of the participant's midterm score was tallied (see Table 4-16).

Table 4-16 Problem 4T Problem-Solving Frequencies

| **Problem-Solving Domain** | **Frequency** |
|---|---|
| **Checking** | 8 |
| **Executing** | 11 |
| **Justifying** | 20 |
| **Orienting/Sense-Making** | 10 |
| **Planning** | 13 |
| **Representing/Connecting** | 6 |
| **Reviewing** | 11 |
| **Total** | **79** |

As seen in Table 4-16, the problem-solving strategy of justification was used with the most frequency while solving Problem 4T, in contrast, representing/connecting was the least utilized problem-solving strategy while solving Problem 4T.

To determine if problem-solving strategy identification varies widely among the participant midterm score groups the data was disaggregated to explore this (see Table 4-17). Table 4-17 shows the frequency of the use of the problem-solving strategies by Midterm 2 RRC problem score group on Problem 4T.

Table 4-17 Problem 4T Problem-Solving by Midterm 2 RRC Problem Score Group

| | **Developing** | **Proficient** | **Exemplary** |
|---|---|---|---|
| **Checking** | 0 | 2 | 6 |
| **Executing** | 1 | 4 | 6 |
| **Justifying** | 3 | 3 | 14 |
| **Orienting/Sense-Making** | 1 | 4 | 5 |
| **Planning** | 2 | 5 | 6 |
| **Representing/Connecting** | 1 | 1 | 4 |
| **Reviewing** | 1 | 4 | 6 |
| **Frequency** | 9 | 23 | 47 |
| **Frequency per Person** | 4.5 | 3.83 | 7.83 |
| **Expected number (based upon uniform distribution 79/14 per person)** | 11.29 | 33.86 | 33.86 |

As seen in Table 4-17, the use of problem-solving strategies was utilized the most by the *exemplary* group, followed by the *proficient* group, and the *developing* group utilized problem-solving domains the least while solving Problem 4T. With a total number of 79 instances of problem-solving codes across all 14 participants, if we assume that this would be uniformly distributed across all participants the expected number of instances for each Midterm 2 RRC problem score group shows the *developing* group and the *proficient* group instances are lower than expected.

Table 4-18 shows some example excerpts of the problem-solving coding of the task-based interview transcripts of Problem 4T.

Table 4-18 Sample Excerpts by Problem-Solving Strategy Code Problem 4T

| | |
|---|---|
| **Checking** | *Penny*: "It doesn't look right, so I'm going to do it over."<br><br>*Percy*: "No, I did dump something definitely wrong. Wait, I did do something wrong."<br><br>*Earl*: "And we know that feet should be below. So, we can do 4 times 9 pi feet squared. Cancel out the feet." |
| **Exec.** | *David*: "So, take the derivative of the volume formula"<br><br>Elsa: "Then I get that as the volume, and then I'm just going to derive that using the 1/12 pi basically outside so that it will, since it's a constant multiple." |
| **Justifying** | *David*: "Since we're looking for solving for the heights. So, we're gonna use our height equals. 6 over 12. Solve for a r, you got to multiply both sides by h. So, r equals. 1/2 h."<br><br>Penny: "… to find my second cone, I'm just going to go ahead and do this again the same way I did it before. which is just solving for R2. And then I got R2 equals three over two feet. And then solving for my second one. I would do the same thing and plug them all in two times dHdT, but we don't have that." |
| **Orienting/ Sense-Making** | *Eve*: "…Height of water is 3 feet I'll use it similar triangles, okay So The volume is 1 by 3 pi r squared h. r by h is equals to 6 by 12, which is 1 by 2. r by h is equals to 1 by 2. So, then r is equals to h by 2. Plugging this in over here."<br><br>*Pat*: "Okay, I'm gonna draw my little diagram. Radius is six feet. The height of the whole thing is 12. dv dt is equal to negative 2 feet cubed per second. Okay. What is the rate of change of the water depth when the water depth is three feet?" |
| **Planning** | *Donald*: "In this case we're looking at the h or the rate of change the water depth is draining. So, I have to do. So, the derivative of the rate of change part thing is."<br><br>*Elsa*: "then I have the water depth that I am using for this problem because I want to find the rate of change of the water depth whenever it is at three feet so then I'm going to plug that in." |
| **Rep/Con.** | *Echo*: "So, okay. So, I need to look, similar triangles. We've got the side, so that's the relationship between the height and the radius. So, there's a relationship between the height and the radius."<br><br>*Earl*: "So now we take the derivative." |
| **Reviewing** | *Donald*: "Trying to retrace my steps a little bit." Donald<br><br>*Penny*: "It doesn't look right, so I'm going to do it over." Penny<br><br>*Echo*: "And I know this is on the right track because we're looking for change in water depth, which is dh over dt. And that's the only variable we have remaining." |

*4.3.6 Comparing Problem-Solving on Phase 2 Problems*

As described in the Methodology, Phase 2 of the task-based interview consisted of two related rates of change problems. The two problems were almost identical and only differed by the numerical values presented in the problems. Problem 3N was an online problem, and Problem 4T was a traditional textbook paper-and-pencil problem. Table 4-19 shows the frequency of use of the problem-solving strategies for each participant for Phase 2 of the task-based interview.

Table 4-19 Phase 2 Problem-Solving Frequency by Participant

| Student | Problem 3N | Problem 4T |
|---|---|---|
| **Echo** | 8 | 8 |
| **Eboy** | 3 | 2 |
| **Earl** | 15 | 13 |
| **Ed** | 1 | 6 |
| **Elsa** | 10 | 11 |
| **Eve** | 3 | 7 |
| **Pamela** | 1 | 4 |
| **Paris** | 1 | 4 |
| **Pat** | 4 | 5 |
| **Penny** | 8 | 5 |
| **Percy** | 0 | 4 |
| **Peter** | 0 | 1 |
| **David** | 4 | 4 |
| **Donald** | 0 | 5 |

As seen in Table 4-19, for 9 out of 14 participants, I identified more instances of problem-solving strategies on the traditional format problem 4T than for the online problem 3N.

*4.3.7 Textbook vs. Online Problem-Solving*

A comparison of the problem-solving strategy use by each participant on the textbook (paper-and-pencil format) problems, Problems 1T and 4T, and the online problems, Problems 2N and 3N, are given on Table 4-20.

Table 4-20 Textbook vs Online Problem-Solving Use by Participant

| | Problems 1T and 4T | Problems 2N and 3N |
|---|---|---|
| **Echo** | 14 | 12 |
| **Eboy** | 6 | 6 |
| **Earl** | 35 | 24 |
| **Ed** | 11 | 4 |
| **Elsa** | 23 | 23 |
| **Eve** | 12 | 6 |
| **Pamela** | 9 | 1 |
| **Paris** | 12 | 5 |
| **Pat** | 12 | 8 |
| **Penny** | 13 | 16 |
| **Percy** | 9 | 0 |
| **Peter** | 9 | 0 |
| **David** | 8 | 9 |
| **Donald** | 13 | 2 |
| **Total Frequency** | 186 | 116 |

As seen in Table 4-20, when looking at the total number of instances of problem-solving strategies identified in the interviews, I identified 70 more instances of problem-solving strategies on the traditional problems than for the online problems. The data also shows that Eboy and Elsa used the same number of problem-solving strategies while solving both textbook and online problems while Echo, Earl, Ed, and Eve used more problem-solving strategies while solving the traditional paper-and-pencil problems. All participants in the proficient group, except for Penny, used more problem-solving strategies on the traditional problems and Percy and Peter used none on the online problems. For the developing group, David's use of problem-solving strategies was slightly higher on the online problem, but Donald's was much lower for the online problem.

To contrast the problem-solving strategy frequencies identified when participants solved the textbook problems (1T and 4T) versus when they solved the online problems (2N and 3N), frequencies were tallied for each problem-solving domain (see Table 4-21).

Table 4-21 Textbook vs. Online Problem Solving by Domains

| Problem-Solving Domain | Textbook | Online |
|---|---|---|
| **Checking** | 16 | 9 |
| **Executing** | 24 | 19 |
| **Justifying** | 41 | 21 |
| **Orienting/Sense-Making** | 26 | 23 |
| **Planning** | 32 | 23 |
| **Representing/Connecting** | 23 | 11 |
| **Reviewing** | 24 | 10 |

The tallies in Table 4-21 show that participants used the problem-solving domains of justifying and planning with the most frequency when solving the textbook problems, and the problem-solving domains of orienting/sense-making and planning with the most frequency when solving the online problems of the task-based interviews. The frequency of justifying, representing/connecting, and reviewing on the textbook problem was approximately double that for these same domains on the online problem. However, the initial phase of orienting/sense-making had similar frequencies between the problems.

## 4.4 View an Example Feature

Online homework platforms offer many features to help students learn concepts and scaffolds to help them solve problems. The online homework platform used in this study offers a 'view an example' feature that shows a similar problem with different numbers than the given problem. The transcripts of the task-based interviews were coded and during the coding three uses or themes emerged for the participants using the 'view an example' feature. As mentioned in Section 3.5, participant use of the 'view an example' feature entailed: mimic, process, and sense-making. When a student uses the 'view an example' feature, a similar problem is shown to the student with an example similar to the original problem that varies the specific numerical data associated with the problem situation (See Figure 4-10). The first code, mimic, corresponds to a participant's use of 'view an example' to mimic the example in solving the problem at hand (i.e., they follow the given example and substitute their problem's numbers into the example problem). Participants could use the 'view an example' feature to mimic at any point while working on problems 2N and 3N. Participants' problem-solving strategies were coded up until they chose to use the 'view an example' feature to mimic because once they began to mimic the example they no longer were engaged in robust problem solving behaviors. Percy, Peter, and Donald chose the 'view an example' feature to mimic at the onset of working on Problem 3N without exhibiting any use of problem-solving strategies. Other participants such as Ed, Pamela, and Paris, exhibited the use of orienting/sense-making before using the 'view an example' feature to mimic.

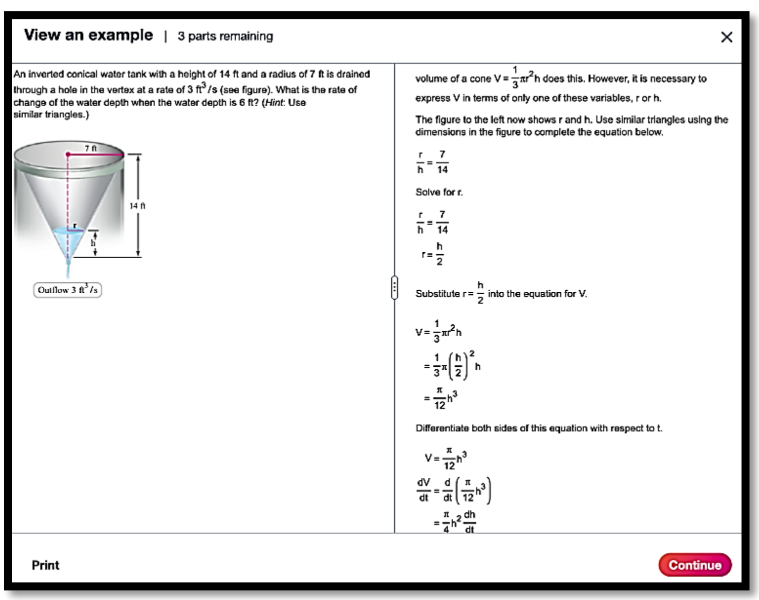



Figure 4-10 'View an Example' Screenshot

The second code, sense-making, corresponds to a participant's use of 'view an example' to try to gain an understanding about how to solve the problem correctly. The third emergent code, process, corresponds to a participant's use of 'view an example' after having solved the problem and review the example to understand when and where they made a mistake solving the problem. Problem 2N and Problem 3N were the two online problems in the task-based interview that participants had the chance of using the 'view an example' feature. Table 4-22 shows some examples of the participant interview data that exemplify the emergent codes for use of the 'view an example' feature of the online homework platform. Note that participants who mimic the example begin following it right away, those who use the example for process view the example after they finish the problem and have questions about their answers, and those who use the example for sense-making use the example to help them determine how to approach the problem or for clarification.

Table 4-22 Selected Excerpts 'View an Example' Feature Use by Code

| **'View an Example' Emergent Code** | ***Sample Excerpts*** |
|---|---|
| **Mimic** | *Paris*: "So then I started following the example and plugging in my numbers"<br><br>*Donald*: "Um, did this one, I was honestly going with the way the view my example showed. The view and example, so I did exactly those steps, but just different numbers with my, and I mixed in my numbers. I put in my numbers with it." |
| **Process** | *Earl:* "So problems like this whenever I don't understand the answers. That's when I look at, especially that's when I look at the examples." |
| **Sense-Making** | *David*: "So I would use the view example to figure out, because I don't have any idea what the expression is." |

As shown in Table 4-23, each participant's use of the 'view an example' feature is tabulated as they engaged in solving Problem 2N of the task-based interview. For those participants who did not use the ‘view an example’ feature, there is no number recorded (instead an “x” appears in each entry of the corresponding participant row in the table). Since the Table 4-23 tabulates only the use of the ‘view an example’ feature on Problem 2N, each column can only have entries of 1, 0, or “x”. Table 4-23 shows that seven participants used the ‘view an example’ feature. All of those who used the features, used it to mimic the example problem to solve Problem 2N. On problem 2N, none of the participants used the ‘view an example’ for sense-making or for understanding or learning the process.

Table 4-23 ‘View an Example’ Use Codes for Problem 2N by Participant

| | Mimic | Sense-Making | Process |
|---|---|---|---|
| **Echo** | x | x | x |
| **Eboy** | 1 | 0 | 0 |
| **Earl** | x | x | x |
| **Ed** | x | x | x |
| **Elsa** | x | x | x |
| **Eve** | 1 | 0 | 0 |
| **Pamela** | 1 | 0 | 0 |
| **Paris** | 1 | 0 | 0 |
| **Pat** | x | x | x |
| **Penny** | x | x | x |
| **Percy** | 1 | 0 | 0 |
| **Peter** | 1 | 0 | 0 |
| **David** | x | x | x |
| **Donald** | 1 | 0 | 0 |

Problem 3N of the task-based interview was also an online problem where participants could use the ‘view an example’ feature. Table 4-24 shows the tabulations for ‘view an example’ for Problem 3N.

As before, for those participants who did not use the ‘view an example’ feature, there is no number recorded (instead an “x” appears in each entry of the corresponding participant row in the table). In addition, since Table 4-24 tabulates only the use of the ‘view an example’ feature on Problem 3N, each column can only have entries of 1, 0, or “x”. As seen in Table 4-24, three participants did not utilize the ‘view an example’ feature for Problem 3N. It also shows that one participant, Earl, used the ‘view an example’ feature to make sense of the problem. Earl and David used the ‘view an example’ feature to learn the process of how to solve Problem 3N.

Table 4-24 ‘View an Example’ Use Codes for Problem 3N by Participant

| | Mimic | Sense-Making | Process |
|---|---|---|---|
| **Echo** | x | x | x |
| **Eboy** | 1 | 0 | 0 |
| **Earl** | 0 | 1 | 1 |
| **Ed** | 1 | 0 | 0 |
| **Elsa** | x | x | x |
| **Eve** | 1 | 0 | 0 |
| **Pamela** | 1 | 0 | 0 |
| **Paris** | 1 | 0 | 0 |
| **Pat** | 1 | 0 | 0 |
| **Penny** | x | x | x |
| **Percy** | 1 | 0 | 0 |
| **Peter** | 1 | 0 | 0 |
| **David** | 0 | 0 | 1 |
| **Donald** | 1 | 0 | 0 |

Three participants in the task-based interview did not access the ‘view an example’ feature for any of the online problems. Those three participants were Echo, Elsa, and Penny. During the task-based interview, Echo said that she rarely uses the ‘view an example’ feature because it makes it too easy. Her only exception to using it is the due date is fast approaching and she does not have time to finish. Elsa stated that if she misses a problem that it is usually a small error that she can find by going back over her work. Penny’s reason for not using the ‘view an example’ feature is that using her notes is easier and that’s the way her professor taught her and the professor has seen the exam.

## 4.5 Evidence of Transfer

Participants of the task-based interviews showed evidence of using the knowledge that was gained while solving the first problem of each phase to solve the second of that phase. Problems 1T and 2N as well as Problems 3N and 4T are remarkably like each other and besides using different numbers the only difference is the medium by which the questions are delivered. Transfer, as described in Chapter 2 Section 6, is the ability to apply previously learned knowledge or skills to a new situation or problem. This means that transfer can only be applied when solving Problems 2N and 4T in the task-based interview.

Table 4-25 Sample Excerpts Coded as Transfer in Phase 1

| Participant | Evidence |
|---|---|
| **Earl** | "I was trying to go the same problem as last time." |
| **David** | "It's almost the same problem." |
| **Percy** | "Okay, so I was on the right track when I was doing it. That was the, that's, that's the one I was missing. Okay, because you need the founded." |
| **Pat** | "Oh, it's the same thing. So, it was just like the last problem" |
| **Donald** | "…this one is the same process a little bit that it's got to be Pythagorean theorem so that's a squared plus b squared c squared." |
| **Elsa** | "So, this problem was pretty similar to the last problem." |

As seen in Table 4-25, statements that were made by the participants while solving Problem 2N that provides evidence that participants were applying skills that were used in Problem 1T to solve Problem 2N.

In Phase 2 of the task-based interview, participants solved an online problem first and then solve a textbook problem. Participants showed evidence of using the skills learned while solving Problem 3N to solve Problem 4T. Table 4-26 lists the evidence that shows that transfer was used to solve problem 4T.

Table 4-26 Sample Excerpts of the Coded as Transfer in Phase 2

| Participant | Evidence |
|---|---|
| **Pamela** | "Since I just got to see an example, or since I did one before, and I kind of knew the steps already, then I knew the formula for the volume of a cone, which was 1/3 pi r squared times the height." |
| **Peter** | "Okay Okay, so this is basically similar to the problem. I just did." |
| **Earl** | "Yeah, it was the same process last one, but this time I didn't have to help me solve this, but I think doing that online one before helped me a lot, because not... it refreshed my memory on how to do these." |
| **David** | "Based on what I did earlier I know the volume" |
| **Pat** | "Just like the other problem, you have to, so volume is equal to 1 over 3 pi r squared times h." |
| **Eve** | "Okay, so I used the same steps that was provided by the example." |

As seen in Table 4-26, some participants used the skills learned from solving Problem 3N to solve Problem 4T. For some of the participants, like Eve, those skills were learned from using the 'view an example' feature that was utilized while solving Problem 3N which is an online problem. During Phase 1 of the task-based interviews, I identified transfer for five of the 14 participants using transfer from Problem 1T to 2N. Of the 14 participants in the task-based interviews, I identified 11 participants using transfer from Problem 3N to Problem 4T.

# Chapter 5

# Discussion and Conclusion

In this chapter, I will present the key findings from this investigation. Then, I will interpret the results presented in the previous chapter. After the presentation of the key findings and interpretation of the results, I will discuss the limitations of this study and directions for future research.

This investigation aimed to explore how problem-solving strategies differ when solving related rates of change homework problems which include access to a 'view an example' feature versus textbook paper-and-pencil related rates of change problems. In order to guide this investigation, I sought to answer the following research questions:

(1) How do students' problem-solving strategies when working online homework on related rates of change problems compare with their problem-solving strategies when working paper-and-pencil homework related rates of change problems?
(2) What influence does the "view an example" feature in online homework have on a student's problem-solving strategies when working an online related rates of change homework problem?

I chose to investigate problem-solving and related rates of change because of the nature of related rates of change. Related rates of change is typically tested and requires problem-solving. Also, I could not find any research that investigated problem-solving and related rates of change and this is an issue that I wanted to investigate.

## 5.1 Problem-Solving

In this study, four findings emerged from the task-based interviews with respect to participant's use of problem-solving strategies when working related rates of change homework problems. First, more instances of problem-solving strategy use were identified when participants worked paper-and-pencil homework on related rates of change problems than when they use working online homework on related rates of change problems. Second, more instances of the use of problem-solving strategies from the *exemplary* group were identified than for the other two Midterm 2 score groups. Third, when participants used the ‘view an example’ online homework feature, fewer instances of problem-solving strategy use were identified. Lastly, four uses emerged on participant use of the ‘view an example’ feature: those who mimic the example, those who use the example to learn the process, those who use the example for sense-making, and those who do not use the ‘view an example’ feature. In the following sections, I delineate the four findings and how it relates to existing research.

*5.1.1 Online Versus Paper-and-pencil RRC Problems*

The first finding that emerged from this investigation was that the data suggested that participants used more problem-solving strategies on RRC problems when working paper-and-pencil homework on related rates of change problems than they used when working online homework on related rates of change problems. As seen in Table 4-20, the frequency of problem-solving strategies identified was higher for the paper-and-pencil homework problems, but this varied by the participants’ Midterm 2 RRC problem score. This may be explained by the fact that the ways in which students used the online features reduced the problem into a non-problem for the problem solver. That is, it is no longer a problem as defined by Lester (Lester, 2013, as cited in Álvarez et al., 2018, p. 233) and seems to contradict Dorko’s (2020b) assertion that students problem-solving in the same manner as a mathematician. According to Dorko (2020b) and Carlson and Bloom (2005) a mathematicians’ work would include orienting, planning, executing, and checking. Overall, the participants in this investigation used all problem-solving domains more when working paper-and-pencil RRC problems.

5.1.1.1 Paper-and-pencil RRC Problems

On Problems 1T and 4T, the traditional paper-and-pencil RRC problems, participant problem-solving pathways aligned with Carlson and Bloom's (2005) framework for problem-solving. That is, participants exhibited the problem-solving phases of orienting, planning, executing, and checking when working the paper-and-pencil problems as seen in the descriptions of the task-based interviews of Echo, Eboy, and Earl (or, in Sections 4.2.1, 4.2.2, and 4.2.3) in particular. Problems 1T and 4T of the task-based interview were traditional paper-and-pencil related rates of change problems from the textbook. Since Problems 1T and 4T were traditional paper-and-pencil related rates of change homework problems most participants followed the framework of Carlson and Bloom (2005) Multidimensional Problem-Solving Framework while working the problem. Participants exhibited the cyclic nature of problem-solving while working on Problem 1T of the task-based interview. Most participants exhibited the problem-solving phases of orienting, planning, executing, and checking when working the paper-and-pencil problems.

On Problem 1T of the task-based interview, participants showed instances of using problem-solving strategies 107 times while working on the problem. There were 79 instances of participants showing use of problem-solving strategies while working Problem 4T. Problem 4T was a paper-and-pencil related rates of change homework problem and participants exhibited similar behavior while working Problem 4T as they did on Problem 1T. There was a decrease in the use of problem-solving strategies between 1T and 4T. This decrease could possibly be attributed the order that the problems were solved during the task-based interviews. Problem 1T was a novel problem to participants during the task-based interview and Problem 4T was similar to Problem 3N that participants had just attempted to solve so that novelty was lost on Problem 4T. While working Problem 4T, participants exhibited quantitative reasoning in a transfer situation as asserted by Lobato and Siebert (2002). This was exhibited by participants like Earl who stated, “Yeah, it was the same process last one, but this time I didn't have to help me solve this, but I think doing that online one before helped me a lot, because not... it refreshed my memory on how to do these.” This use of quantitative reasoning in a transfer situation was also exhibited when participants solved 2N after working 1T.

Problem-solving strategy use in the domains of planning and justifying were exhibited with the highest frequency on the paper-and-pencil related rates of change problems (see Table 4-21). This could be attributed to the online homework features that require less planning because of the way that the problems are scaffolded. This also may be due to the problems being on the online platform and the participants not feeling the need to explain their reasoning. Planning and justifying included participants accessing their conceptual knowledge (Álvarez et al. 2019; Carlson & Bloom, 2005) and was exhibited with the highest frequency by the *exemplary* group. This may be due to the fact that the exemplary Midterm 2 RRC group is comprised of participants with the highest conceptual (process) scores among all participants as shown in Table 4-7 and Table 4-17.

The use of orienting/sense-making differs the least between textbook and online problems. The standard procedure for solving related rates of change problems that most of the participants were taught was to start by drawing a diagram. Engelke (2007) found that drawing a diagram is the first phase for solving related rates of change problems. Since drawing the diagram is part of the orienting/sense-making phase this is possibly why these counts were similar. Therefore, in these types of problems, it makes sense that orienting/sense-making would have similar frequencies regardless of the modalities that the related rates of change problems are presented.

#### 5.1.1.2 Online Related Rates of Change Problems

Problems 2N and 3N were online related rates of change homework problems and participants had the option of using the ‘view an example’ feature. While working

Problem 2N, seven participants used the ‘view an example’ feature to mimic the example problem at some point while working on the problem. Mimicking the example problem is one of reasons that Dorko (2021) found that students use the ‘practice another version’ help feature found in online homework systems. When a participant uses the ‘view an example’ feature to mimic, I observed that they no longer engaged in problem-solving strategies, partly due to the fact that mimicking a solution changes the nature of the task and their engagement with the task. This accounts for the low number of instances of problem-solving identified for problems 2N and 3N (see Table 4-20). This appears to contradict Dorko’s (2020b) finding that student activity when solving online homework problems is cyclic and similar to what a mathematician would do when problem-solving.

On Problem 2N of the task-based interview, I identified 58 instances or interview episodes of problem-solving strategy use. Similarly, participants showed 58 instances of using problem-solving strategies while working Problem 3N. At some point while working Problem 3N, nine participants (e.g., Pamela and Percy) used the ‘view an example’ feature to mimic the example. Similar to the observation for Problem 2N, I observed that these participants no longer engaged in problem-solving strategies once they used the online feature, partly due to the fact that mimicking a solution changes the nature of the task and their engagement with the task. Problem 3N involved an auxiliary problem that needed to be solved and most participants had difficulty solving it. This observation is similar to the findings of Engelke (2004, 2008), Martin (2000), and Mkhatshwa’s (2020) that the inclusion of an auxiliary problem increases the difficulty for students and makes arriving at the correct solution less likely. The inability to solve the auxiliary problem in Problem 3N led to many participants using the ‘view an example’ feature.

On the online problems 2N and 3N, I found that participants used the problem-solving domains of justifying, planning, and checking less frequently  than they did on the paper-and-pencil problems. Moreover, some participants did not use these domains at all while working on the online problems (see Table 4-12 and Table 4-19). The use of the ‘view an example‘ feature to mimic the example problem appears to result in the decrease of use of problem-solving strategies in all RRC Midterm scoring groups and all participants that use the ‘view an example’ feature. The use of the ‘view an example’ feature led to participants relying on the example problem to mimic the solution. This reliance hindered participants from having to justify their work, plan their next steps, and check their solutions because the example problem did all of this for them. The multiple attempts that are given in online homework may have influenced the guessing behaviors I observed. My observations align with Dorko’s (2018, 2020a) findings that multiple attempts increase guessing behaviors in online homework. In my observations, participants used their multiple attempts to check if their answers were correct. However, it their answer was incorrect, the participants would make small adjustments or guesses to input and check their answers again. Similar to Dorko’s (2020b) findings, I observed participants using the multiple attempts given to them on the online format as a formative assessment. This instant feedback and scaffolding may have led to a decrease in some problem-solving strategy use while working online problems. This may have accounted for the fewer instances of problem-strategy use on the online homework problems versus paper-and-pencil format problems overall (116 compared to 186, respectively).

### *5.1.2 Problem-Solving By Midterm 2 RRC Problem Score Group*

The second finding observed was that more instances of the use of problem-solving strategies from the *exemplary* group were identified than for the other two Midterm 2 RRC problem score groups. Participants in the *exemplary* group used the ‘view an example’ feature less than the other two Midterm score groups. Being less dependent on the ‘view an example’ feature resulted in the *exemplary* group using more problem-solving strategies on Problems 2N and 3N than the other two Midterm score groups. Being less dependent on the use of the ‘view an example’ feature can possibly be attributed to these participants having better content knowledge that contributes to the use of more problem-solving domains as theorized by Álvarez, et al. (2019). It can be presumed that the *exemplary* group had better content knowledge of related rates of change than the other two Midterm 2 score groups since they did score higher on Midterm 2’s related rates of change problem.

In order to be in the *exemplary* group, participant’s Midterm 2 RRC problem scores were in the top quartile in both process points and total points. Being in the top quartile in both process and total points, it may be a reasonable assumption that this group has greater facility with both process and procedures needed for solving related rates of change problems. The other two Midterm 2 score groups, *proficient* and *developing*, were able to deal with the procedures of solving related rates of change problems but lack the needed contextual meaning that would negate the need to use the ‘view an example’ feature. This abstract-apart ideas that some participants displayed could be attributed to their learning how to solve related rates problems by memorizing the steps without context as asserted by White and Mitchelmore (1996). According to White and Mitchelmore (1996), abstract-apart ideas is easier to learn since it is all symbolic. As opposed to abstract-general that is linked to a participant’s conceptual knowledge. This all could be attributed to the reason why the *exemplary* group used all problem-solving domains with greater frequency than the other two Midterm 2 scoring groups on every problem with the exception of orienting on Problem 1T and representing/connecting on Problem 2 where both *exemplary* and *proficient* groups had the same frequencies. Furthermore, all participants were able and allowed to complete all of their problems and presented an answer for each problem.

*5.1.3 Problem-Solving and the 'View an Example' Feature*

The data from the online homework provider for the related rates of change homework for the Spring 2023 showed that on the problems that over 400 students attempted on five of the six problems over 50% of the students used a help feature. Eleven out of the 14 participants self-reported that they did use the 'view an example' feature while completing homework. Two participants self-reported that they have no experience with online homework in a mathematics course and one participant did not answer the question. This leads us to my third finding that emerged from this investigation which was that participant's use of problem-solving strategies decreased when using the 'view an example' feature when working an online related rates of change homework problem. This finding aligns with my first finding that participants used fewer problem-solving strategies when working online related rates of change homework problems (see Table 4-12 and Table 4-19) which appears to contradict Dorko's (2020b) assertion that students engage in problem-solving as would a mathematician when working online problems. According to Dorko's (2020b) study, mathematicians have the tendency to analyze a problem, plan and execute a solution method, verify their results, and if their results are incorrect, they return to analyze the problem. In problem-solving, this means that they would exhibit behaviors of orienting, planning, executing, and checking and if their working is incorrect, they would return to planning. However, for my participants, 11 out of 14 participants chose to use the 'view an example' feature and nine of those 11 choosing to use the feature, used it to mimic the example problem rather than engage in mathematician-style behavior. My observations reveal that as soon as participants began to mimic the example problem, the path to solve the problem becomes known and the “problem” is no longer considered problem as defined by Schoenfeld (2013). Hence, my view that little to no problem-solving occurs thereafter.

*5.1.4 Use of The ‘View an Example” Feature*

The fourth finding that emerged from this investigation was four ways that participants used the ‘view an example’ feature: those who mimic the example, those who use the example to learn the process, those who use the example for sense-making, and those who do not use the ‘view an example’ feature. These four uses aligned with the reasons that Dorko (2021) found that students used in her ‘practice another version’ study which were:  to copy and paste the ‘practice another version’ solution; to view as a template; to see solutions to similar problems to troubleshoot; to check if they were on the right track; to see the steps to solve a problem; and to see the form of the answer. Copy and paste the ‘practice another version’ refers to copying and pasting the example problem’s solution without changing any numerical values. Viewing as a template for the ‘practice another version’ is the same as what I have called mimicking in this investigation. To see the solutions to similar problems to troubleshoot is the same as my emerging code of process where after participants enter an incorrect answer, they use the practice another version to see the process of solving the problem correctly. Checking to see if they are on the right tract and to see the steps to solve a problem would align with the emerging code of sense-making in this investigation. None of the participants in my study used the ‘view an example’ feature to *see the form of the answer*. This may be due to the nature of related rates problems and that participants are familiar with the form of the answers. Furthermore, the online problems give the participants a blank space to fill in the answer with a drop-down box where they choose the units of their answer.

The reason, however, why the participants chose to engage with the 'view an example' feature differs by participant and what they may be trying to achieve at that moment. Earl viewed the example to learn the *process* after missing the problem and stated, "So problems like this whenever I don't understand the answers. That's when I look at, especially that's when I look at the examples." David viewed the example to make sense of how to solve the problem because as he said, "I don't have any idea what the expression is". Many participants, like Donald, stated that they use the 'view an example' feature whenever they are working a word problem. The reason why the participants used the 'view an example' feature, which I found to align with Schoenfeld's (2010) framework, could be a part of their goal of solving the problem correctly, their knowledge of related rates of change problems, their confidence in solving the problem correctly, or based on their decision to get it done as soon as possible. The reasoning could also be based on the participants didactic contract, as defined by Dorko (2020b), that was formed between me as the researcher and the participant's goals and expectations as a part of this investigation. The participants of the study expressed the desire to answer the problems correctly and that desire led to some explaining the 'view the example' feature as it was the own work.

Echo, Elsa, and Penny exhibited problem-solving behaviors similar to those in Carlson and Bloom's (2005) Multidimensional Problem-Solving Framework on all problems regardless of the modality in which they were presented. They exhibited behaviors of orienting, planning, executing, and checking. Echo, Elsa, and Penny also exhibited the cyclic behavior of problem-solving as presented by Dorko (2020b) and Carlson and Bloom (2005), They were the only participants that did not use the 'view an example' feature and their problem-solving use was similar for both problems in each phase.

## 5.2 Limitations

I do recognize that there are limitations to this study. The first limitation of this study was the small sample size of the developing group in comparison to the other groups. Thus, frequency counts for the differing codes may have been skewed or sensitive to the small sample size. Another possible limitation is the nature of the Midterm 2 RRC problem that was used to form the different groups from which to recruit participants. The Midterm 2 RRC problem was heavily scaffolded (e.g., the figure, while not labeled, was provided for students) and some participants with scores in the *exemplary* group may have had scores in other quartiles if drawing a picture or diagram could have been one of the criteria for scoring the problems. The Midterm 2 RRC problem format and the online format which provide diagrams for students poses a problematic issue if one of the goals of the RRC problems is to have students draw a diagram to represent a problem situation (Engelke, 2004; Mkhatshwa, 2019).

Another possible limitation is that while participants were asked to work on the problems to reflect the typical way they approach the homework, the traditional paper-and-pencil format problems in the task-based interview setting possibly limited their tendency to look for written examples in the textbook or online even when the problem was presented in paper-and-pencil format. However, the rationale for the task-based interview format was also intended to gain insight into the problem-solving strategies being developed or practiced while working online homework problems in contrast to the paper-and-pencil format of the Midterm 2 assessment that precludes the readily available features of online homework.

## 5.3 Future Research

Based upon my findings, more research on how problem-solving skills are being developed or circumvented by exclusive or near-exclusive use of online homework platforms in today's mathematics courses. Although Dorko (2020b) found that, students online work is similar to what a mathematician would do when problem-solving, more research needs to be conducted that examines not only whether students are successful in completing a problem (by, say, getting the right answer), but whether they are also developing the intended problem-solving skills associated with doing mathematics. In addition, course assessments, much like the one in this study, are administered in a static paper-and-pencil format so the features of the online platforms and the format of online homework problems may not be adequately building the knowledge needed to solve complex problems on the assessment.

Some changes for future research that I would adopt include allowing participants the option to use resources that they would use at home such as class notes and textbooks when working the problems and refining the procedures for assigning participant score groups.

## 5.4 Conclusion

While the benefits of online homework platforms such as immediate feedback, multiple attempts, and hints (Dorko, 2021) may be producing positive outcomes in student success, it is unclear at best whether both formats provide similar opportunities for the development of problem-solving strategies from which to build further mathematical knowledge. For participants in the *proficient* and *developing* groups, there were marked declines in alignment with the problem-solving domain codes used in this study when working online problems and using the 'view an example' feature. While the online features allow for immediate feedback and, perhaps, align with students' didactic contract (Dorko, 2020b) of achieving the correct answer to the problem, my participant data suggests that problem-solving is diminished when using these features. Thus, for some instructors, the didactic contract of developing students' capacity for problem-solving is unmet. In addition, as seen in Paris, Peter, and Eve's coding (see Figure 4-3 and Figure 4-5), the problem-solving reviewing domain may be particularly influenced or diminished by the instant feedback received. The scaffolding of the problem could also lead to participants not planning on how to solve an online problem since the steps are already planned out for them to solve the problem. All participants had previously worked Problems 2N and 3N as part of their related rates online homework assignment in the course several weeks prior to the task-based interview.

In this setting, we identified more instances of problem-solving strategy use when participants engaged in the paper-and-pencil format related rates of change problems. This may suggest that the online homework platform features may influence this use. In addition, instances of problem-solving strategy use by participants from the *exemplary* Midterm 2 RRC problem score groups were higher than for those with scores in other quartiles represented. While this may have been expected, the data indicate that there could be some aspects of the online homework 'view an example' feature that students who have more difficulty with computations or the process (e.g., participants in the developing score group) may more readily rely on this feature and circumvent opportunities to engage in mathematical problem-solving. Related to this, the four uses—for mimicking, for understanding process, for sense-making, and non-use of the feature—that emerged on participant use of the 'view an example' feature raise important questions about ways in which students can be guided or problems can be restructured to meet both the demand for immediate feedback but also the learning goal of developing as a problem solver by providing scaffolds that encourage sense-making, planning, and justification in a manner that keeps the "problem" in "problem."

## Appendix A

## Consent Form and Demographic Survey

UNIVERSITY OF TEXAS ARLINGTON

My name is Tyson Bailey, and I am asking you to participate in a UT Arlington research study titled, "An investigation of calculus students' problem-solving strategies when working related rates of change problems appearing in online homework versus similar problems in a paper and pencil format." This research study is about how problem-solving strategies may or may not differ when solving related rates of change problems online versus solving related rates of change problems with pencil and paper. You can participate in this research study if you are at least 18 years old.

If you decide to participate in this research study, you will fill out this survey. You will be consenting to the investigators being able to access your MyLabs homework grades, data on your usage of "show an example" in MyLabs, and your related rates of change problems on Midterm 2. The data will be used to understand how your usage of the online homework and its features correlate with your performance on similar exam questions. After the second midterm, a representative subset of twenty-one (21) students will be asked to participate in a one (1) hour task-based interview. For those who participate in interviews, the audio-video recorded task-based interview will consist of solving two (2) online related rates of change problems and two (2) paper and pencil related rates of change problems.

You probably won't experience any personal benefits from participating beyond, possibly, having more exposure and practice to related rates problems. There is a potential risk of loss of privacy and confidentiality. To minimize this risk, the study team will use UTA-approved software to store all research data. All artifacts collected and transcripts of interviews will have identifying information redacted. Each

Intro/Consent

QuestionPro

and transcripts of interviews will have identifying information redacted. Each interviewed participant will be given an appropriate pseudonym, and any reporting of the findings will refer to the participants by their pseudonym. Collection of the interview data (that is, the interview recording and artifacts produced by the subject during the interview process) will be completed in the private office of a researcher. Any electronic data (both the interview recordings and electronic versions of physical artifacts) will be stored on a secure service (UTA OneDrive).

Participants that complete the initial surveys will be given a homework grade of 100 that can be used as a replacement grade for the lowest homework grade of the semester. Alternatively, you may complete the attached problem set to also gain the homework grade if you do not want to participate in the study. No financial compensation will be given for participating in the initial survey. However, participants that complete the task-based interview will be given a $25 Walmart gift card after completing the interview. The Internal Revenue Service (IRS) considers all payments made to research subjects to be taxable income; this may require additional information to be collected from you for tax purposes, such as your social security number.

There is a potential risk of loss of privacy and confidentiality as a risk of participating in this study. The research team is committed to protecting your rights and privacy as a research subject. We may publish or present results, but your name will not be used. While absolute privacy and confidentiality cannot be guaranteed, the research team will make every effort to protect the confidentiality of your records as described here and to the extent permitted by law.

If you have questions about the study, you can contact me at tyson.bailey@uta.edu or my advisor Dr. James Alvarez at james.alvarez@uta.edu. For questions about your rights or to report complaints, contact the UTA Research Office at 817-272-3723 or regulatoryservices@uta.edu.

You are indicating your voluntary agreement to participate by signing on the line below and providing your UTA email address to receive future correspondence regarding the study. Your signature below also gives your consent for release of student data as required by FERPA.

Contact Information

First Name

Last Name

UTA Email Address

Date

By typing your full name it serves as your electronic signature

Gender (optional)

Race (optional)

What year did you graduate from high school or earn GED?

- 2022
- 2021
- 2020
- Before 2020

Indicate your Course Section:

- Math 1426-050 Alice Lubbe MoWeFr 2:00-2:50
- Math 1426-088 Jeremy Glass
- Math 1426-089 Jeremy Glass
- Math 1425-150 Patrick McCormick MoWe 1:00-2:20
- Math 1426-200 Edward Wilson MoWeFr 10:00-10:50
- Math 1426-250 Patrick McCormick TuTh6:00-7:20
- Math 1426-300 Erica Bajo Calderon TuTh 9:30-10:50
- Math 1426-350 Mark Krasij MoWe 5:30-6:50
- Math 1426-400 Mark Krasij TuTh 12:30-1:50

Did you take Math 1426 at UTA in Fall 2022 and if so what was your final grade?

- A
- B
- C
- D
- F
- I did not take Math 1421 in Fall 2022

What is your current classification?

- First Semester Freshman
- Second Semester Freshman
- Sophomore

- Junior
- Senior

What Math courses have you taken previously at the University level?

How many times have you enrolled in Math 1426?

- This is my first time enrolling in Math 1426.
- This is my second time enrolling in Math 1426.
- I have enrolled in Math 1426 three or more times.

Have you used online homework in a previous math course?

- Yes
- No

When doing homework online how often do you use the "show an example" feature?

- very often
- often
- not very often
- never
- I have not used online homework in a previous course.

Appendix B

Alternative Assignment

Name:
Email:
Course Number:

**An investigation of calculus students' problem-solving strategies when working related rates of change problems appearing in online homework versus similar problems in a paper and pencil format.**
**Calculus I**

**Tyson Bailey**
**tyson.bailey@uta.edu**

## Alternative Assignment Calculus I

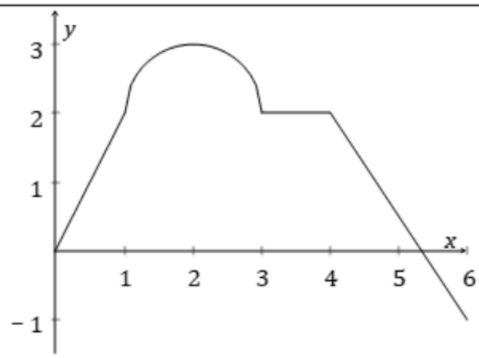
You may receive the 100 homework grade by completing the problems below rather than participating in the research study. If you wish to receive credit for this assignment, please turn in your solutions with work shown and answers clearly written, on your own paper within one week. You must complete all five (5) problems to receive credit.

**Problem 1:**

Assume F is any quadratic function of the form $F(x) = ax^2 + bx + c$.

1. Determine if the following inequality is true or false for all real numbers $v$ and $w$:
$$F\left(\frac{v+w}{2}\right) < \frac{F(v)+F(w)}{2}$$
2. Provide justification of your answer in at least two different ways.
3. Explain your mathematics strategies when starting this problem.

**Problem 2:**




1. Given the above graph of $f(x)$ with domain [0,6], graph the transformed function $\frac{1}{3}|f(2(x-1))| + 4$ for the appropriate domain.

2. What strategies did you use for graphing the transformed function?

**Problem 3:**

1. $2\sin(3x) = \frac{1}{3}$

   (a) Solve the above equation in at *least two different ways*.

   (b) What is your usual approach to solving an equation like this? Explain.

   (c) Explain the advantages and/or disadvantages of each method used.

2. $2x^3 + x^2 + 6 = 13x$

   (a) Solve the above equation in at *least two different ways*.

   (b) What is your usual approach to solving an equation like this? Explain.

   (c) Explain the advantages and/or disadvantages of each method used.

3. $(n^2 - 5n + 5)^{(n^2-11n+30)} = 1$

   (a) Solve the above equation in at *least two different ways*.

   (b) What is your usual approach to solving an equation like this? Explain.

   (c) Explain the advantages and/or disadvantages of each method used.

**Problem 4:**

---

The volume of a sphere is given by the formula $V(r) = \frac{4}{3}\pi r^3$ and the surface area of a sphere is given by the formula $S(r) = 4\pi r^2$.

1. Provide a function that determines how the volume of a sphere changes with respect to the surface area of a sphere. What is the inverse of this function, and what information does the inverse function tell us?
2. Explain your reasoning for determining your solution.
3. Explain in a few sentences the importance of being able to work with multiple functions at once.

**Problem 5:**

Consider the following sums:

1. For n element of the Natural numbers,

$$S_n = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} + \ldots + \frac{1}{n\times(n+1)} \text{ ?}$$

   (a) What is the sum when n = 3? n = 5? What happens for large n?

   (b) Find a formula that gives the sum for any n.

2. For n element of the Natural numbers,

$$S_n = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \ldots + \frac{n}{(n+1)!}$$

   (a) What is the sum when n = 3? n = 5? What happens for large n?

   (b) Find a formula that gives the sum for any n.

3. What strategies did you use to help you find each of the formulas?
4. Explain what you understood about the problem, and explain what questions you still have.

Appendix C

Mathematical Problem-Solving Rubric

22. The base of a right triangle is increasing at a rate of 2 meters per second and the height of the right triangle is increasing at a rate of 12 meters per second.

(a) At what rate is the <u>hypotenuse</u> of the triangle changing at the instant when the base has length 4 meters and the height is 3 meters? Label the units of your solution. {5 pts}

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(4)(2) + 2(3)(12) = 2(5)\frac{dz}{dt}$

$16 + 72 = 10\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{88}{10}$

$\frac{dz}{dt} = 8.8 \ m/s$

1 pt for formula

1 pt correct formula

1 pt take derivative

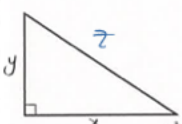
3 pt correctly taking derivative

Up to 6 pts Use of given info (*)

1 pt Solve for dz/dt

1 pt correctly solve for dz/dt

1 correct units




$4^2 + 3^2 = z^2$

$z^2 = 25$

$z = 5$

1 pt label picture

(*) 1 pt for using 4m, 1 pt for using 2 m/sec, 1 pt for using 3m, 1 pt for using 12 m/sec, 1 pt for finding 5m, 1 pt for using 5m

(b) At what rate is the <u>area</u> of the right triangle changing at the instant when the base has length 4 meters and the height is 3 meters? Label the units of your solution. {5 pts.}

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dy}\cdot y + \frac{1}{2}x\frac{dy}{dt}$

$\frac{dA}{dt} = \frac{1}{2}(2)(3) + \frac{1}{2}(4)(12)$

$\frac{dA}{dt} = 3 + 24$

$\frac{dA}{dt} = 27 \ m^2/s$

1 pt formula

2 pt correct formula

1 pt take derivative

5 pt correctly take derivative

Up to 6 pts Use of given info (*)

1 solve for dA/dt

1 correctly solve for dA/dt

1 correct units

(*) 1 pt for using 2 m/sec, 1 pt for using 3 m, 1 pt for using 12 m/s, 1 pt for using ½ with 4m, 1 pt for using ½ with first factor, 1 pt for using ½ with the 2nd factor

Engelke's "A Framework for the Solution Process for Related Rates Problems" is the basis of what I am aligning my rubric for scoring of midterm 2 related rates question.

| | |
|---|---|
| Drawing a diagram<br>• Diagram labeled | 1 point for labeling diagram |
| Constructing Meaningful Functional Relationships<br>• Algebraic equation to relate the variables in the diagram | 1 pt for using a formula<br>1pt for using the correct formula |
| Relate the Rates<br>• Differentiated algebraic equation<br>• Chain rule equation | 1 pt for taking the derivative<br>3 pts for correctly taking the derivative<br>2pt Chain rule (b) |
| Solve for the Unknown<br>• Algebraic manipulations of the differentiated equation | 6 pts for substituting in known values<br>1 pt Solve for dz/dt<br>1 pt for correctly solving |
| Check the Answer for Reasonability<br>• Notation of units | 1 pt Notation of units |

Points are based on activity and not correctness.

Appendix D

IRB Approval

12/16/2022

**IRB Approval of Minimal Risk (MR) Protocol**

**PI:** Tyson Bailey
**Faculty Advisor:** James Mendoza Alvarez
**Department:** Mathematics
**IRB Protocol #: 2023-0109**
**Study Title:** *An investigation of calculus students' problem-solving strategies when working related rates of change problems appearing in online homework versus similar problems in a paper and pencil format.*
**Effective Approval: 12/16/2022**

**Protocol Details**

- Original Protocol Approval Date: 12/16/2022

The IRB has approved the above referenced submission in accordance with applicable regulations and/or UTA's IRB Standard Operating Procedures. The IRB team reviewed and approved this non-federally funded, non-FDA regulated protocol in accordance with the UTA IRB Internal Operating Procedures. The study is approved as Minimal Risk.

**Principal Investigator and Faculty Advisor Responsibilities**

All personnel conducting human subject research must comply with UTA's IRB Standard Operating Procedures and RA-PO4, Statement of Principles and Policies Regarding Human Subjects in Research. Important items for PIs and Faculty Advisors are as follows:

- **Notify Regulatory Services of proposed, new, or changing funding source**
- Fulfill research oversight responsibilities, IV.F and IV.G.
- Obtain approval prior to initiating changes in research or personnel, IX.B.
- Report Serious Adverse Events (SAEs) and Unanticipated Problems (UPs), IX.C.
- Fulfill Continuing Review requirements, if applicable, IX.A.
- Protect human subject data (XV.) and maintain records (XXI.C.).
- Maintain HSP (3 years), GCP (3 years), and RCR (4 years) training as applicable.

Appendix E

Interview Invitation

*An Investigation of calculus students' problem-solving strategies when working related rates of change problems from online homework versus paper and pencil homework.*
Dear [student],

Thank you for consenting, earlier this semester, to be part of this research study in Prof. [ instructor]'s Calculus I class at UTA. **You have been selected to participate in one (1) task-based interview that will last up to one (1) hour**. *For your participation and compensation for your time, you will receive a $25 Walmart gift card at the conclusion of the interview*. In the task-based interview, you will be asked to solve four (4) related rates of change problems. Two (2) of the problems will be online and two (2) problems will be from the textbook. Students chosen for interviews were chosen with the intent of composing a representative sample of all students in the course.

As described in the consent document you signed, the purpose of this project is about problem-solving strategies used when working related rates of change problems from online homework versus problem-solving strategies used when working paper and pencil related rates of change problems. Results of this study will be used to inform my (Tyson Bailey) dissertation study.

As mentioned above, your interview will last no more than one hour and does not require any preparation on your part. The interview will be recorded, as described in the consent form to allow the researchers to review the interactions. Any publication or sharing of the video will protect your identity through methods including but not limited to transcription or reenactment.

Your participation in the interview is voluntary. You may choose not to participate with no penalty to you. Participation will not affect your course grade.

If you have any questions about the interview, you can email me Tyson.bailey@uta.edu. **To participate in the interview,** *please reply to this email* and I will confirm a time as soon as I hear back from you. The interviews will occur in Pickard Hall office 430.

Thank you again for considering this request to participate in an interview,
Tyson Bailey
Graduate Teaching Assistant
UTA Mathematics Department

Appendix F

Interview Protocol

Research Questions:

An investigation of calculus students' problem-solving strategies when working related rates of change problems from online homework versus paper and pencil homework.

1. How do students' problem-solving strategies when working online homework on related rates of change problems compare with their problem-solving strategies when working paper and pencil homework related rates of change problems?
2. What influence does the "view an example" feature in online homework have on a students' problem-solving strategies when working an online homework problem? In what ways do students compensate for not being able to "see another example" when working paper and pencil related rates of change problems?

Task based interview:

- Introduction and Instructions:

Thank you for participating in this investigation of calculus students' problem solving strategies when working related rates of change problems . Today you will be asked to solve four related rates of change problems. The first phase you will solve one pencil and paper problem followed by an online problem. The second phase you will solve one online problem then you will solve a paper and pencil problem. I will be asking you questions about your methods of solving the problems throughout today's session.

I ask that you think aloud while you are working out the problems and I may ask clarifying questions while you are working. For example, if I were working a problem like 2x + 4 = 8 then thinking aloud as I worked would look like this: " I am going to subtract 4 from both sides of the equation to isolate the variable, next I would divide both sides of the equation by 2 to isolate x".

Phase 1 Questions:

**23. Time-lagged flights** An airliner passes over an airport at noon traveling 500 mi/hr due west. At 1:00 P.M., another airliner passes over the same airport at the same elevation traveling due north at 550 mi/hr. Assuming both airliners maintain their (equal) elevations, how fast is the distance between them changing at 2:30 P.M.?

ates | Question 8, 3.11.23 Part 3 of 3 | HW Score: 0%, 0 of 14 points | Points: 0 of 1 | Save

An airliner passes over an airport at noon traveling 530 mi/hr due west. At 1:00 p.m., another airliner passes over the same airport at the same elevation traveling due north at 580 mi/hr. Assuming both airliners maintain their (equal) elevations, how fast is the distance between them changing at 2:00 p.m.?

The equation relating the horizontal distance between the first airliner and the airport, a, the horizontal distance between the second airliner and the airport, b, and the horizontal distance between the two airliners, c is $a^2 + b^2 = c^2$.

Differentiate both sides of the equation with respect to t.

$$(2a)\frac{da}{dt} + (2b)\frac{db}{dt} = (2c)\frac{dc}{dt}$$

(Do not simplify.)

At 2:00 p.m., the distance between the airliners is changing at a rate of about 743.4 mi/hr.

(Round to the nearest tenth as needed.)

1. Have you seen this type of question before today?
    a. Researcher says: Please take a few moments to look at this problem and then begin working it when you're ready. Remember that I'd like you to think aloud as you work on the problem.
        i. If they are writing and they have not spoken out loud I will prompt or ask, "tell me more about what you were thinking when you went from "here" to "here".
        ii. If they get through the problem: Researcher says: Now that you're done, walk me through your process. Or can you walk me through your thinking on this problem.
            1. You'll be watching for: sense making, making a plan, checking for reasonableness, etc.
            2. For specifics: Can you tell me more about how you decided to relate the different quantities?
            3. Do you remember any steps your professor asked you to follow when working on these types of problems? What has helped you learn to work with these types of problems?
        iii. If they didn't get through the problem: Researcher says: Tell me where you are having trouble? What do you remember from previous homework or from the professor's lecture? What do you do when you run into a block like this? What is a typical way you approach a related rates problem?

2. When working related rates problems, what can you tell me about your confidence in successfully completing these types of problems. Explain (tell me more about that or you said “x” can you tell me more about that?).
3. What prompted you to “view an example” or see a similar question for the online problem?
4. How does this question compare to what you worked on in class or in lab? [i.e., Would you describe them as similar, different, or not sure? Explain]
5. When you are having trouble solving a related rates problem, what do you do to make progress? If their answer is “I see another example in the online platform”, then ask what you would do if you were not working online (i.e., for textbook homework or on an exam).

6. Can you describe the process of how your professor tells you to solve related rates of change problems?

7. Can you walk me through your process of thinking?

8. What is your process when you are drawing a diagram for related rates of change problems?

Phase 2 Questions:

ates

Question 11, 3.11.36

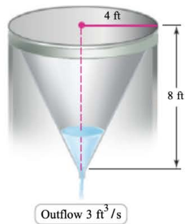
Part 3 of 3

HW Score: 0%, 0 of 14 points

Points: 0 of 1

Save

An inverted conical water tank with a height of 8 ft and a radius of 4 ft is drained through a hole in the vertex at a rate of 3 $ft^3/s$ (see figure). What is the rate of change of the water depth when the water depth is 4 ft? (*Hint*: Use similar triangles.)




Let V be the volume of water in the tank and let h be the depth of the water. Write an equation that relates V and h.

$V = \frac{\pi}{12}h^3$

(Type an exact answer, using $\pi$ as needed.)

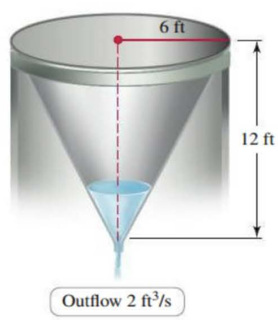
Differentiate both sides of the equation with respect to t.

$\frac{dV}{dt} = \left(\frac{\pi}{4}h^2\right)\frac{dh}{dt}$

When the water depth is 4 ft, the rate of change of the water depth is about $-0.24$ ft/s.

(Round to the nearest hundredth as needed.)

**36. Draining a tank** An inverted conical water tank with a height of 12 ft and a radius of 6 ft is drained through a hole in the vertex at a rate of 2 $ft^3/s$ (see figure). What is the rate of change of the water depth when the water depth is 3 ft? (*Hint:* Use similar triangles.)



3. Have you seen this type of question before today?
    a. Researcher says: Please take a few moments to look at this problem and then begin working it when you're ready. Remember that I'd like you to think aloud as you work on the problem.
        i. If they are writing and they have not spoken out loud I will prompt or ask, "tell me more about what you were thinking when you went from "here" to "here".
        ii. If they get through the problem: Researcher says: Now that you're done, walk me through your process. Or can you walk me through your thinking on this problem.
            1. You'll be watching for: sense making, making a plan, checking for reasonableness, etc.
            2. For specifics: Can you tell me more about how you decided to relate the different quantities?
            3. Do you remember any steps your professor asked you to follow when working on these types of problems? What has helped you learn to work with these types of problems?
        iii. If they didn't get through the problem: Researcher says: Tell me where you are having trouble? What do you remember from previous homework or from the professor's lecture? What do you do when you run into a block like this? What is a typical way you approach a related rates problem?

2 When working related rates problems, what can you tell me about your confidence in successfully completing these types of problems. Explain (tell me more about that or you said “x” can you tell me more about that?).

4. What prompted you to “view an example” or see a similar question for the online problem?

5. How does this question compare to what you worked on in class or in lab? [i.e., Would you describe them as similar, different, or not sure? Explain]

6. When you are having trouble solving a related rates problem, what do you do to make progress? If their answer is “I see another example in the online platform”, then ask what you would do if you were not working online (i.e., for textbook homework or on an exam).

7. Can you describe the process of how your professor tells you to solve related rates of change problems?

9. Can you walk me through your process of thinking?

10. What is your process when you are drawing a diagram for related rates of change problems?

Appendix G

Codebook

## Codebook: A priori and Emergent Codes

**A priori Codes (adapted from Álvarez et al., 2018 and Carlson & Bloom, 2005)**

**Orienting/Sense-making**: The participant identifies key ideas and concepts to understand the underlying nature of the problem. The participant initially engages to make sense of the information given in the problem. i.e., drawing a picture, writing down the given information.

**Planning:** The participant accesses conceptual knowledge and heuristics as a means of constructing, imagining, and evaluating their conjectures. Participants verbalize their strategy and approach to the problem.

**Representing/connecting:** Reformulating the problem by using a representation not already used in the problem or connecting the problem to seemingly disjoint prior knowledge. Participant uses formulas or concepts not given in original problem to solve problem.

**Executing:** The participant accesses their conceptual knowledge, facts, and algorithms when constructing statements and carrying out computations. Participant attempts to solve problem using higher-order techniques.

**Checking:** The participant draws on their conceptual and procedural knowledge to verify the reasonableness of their results and the correctness of their computations. Participant verifies work, checks units, checks reasonableness of solution.

**Justifying:** Communicating reasons for the methods and techniques used to arrive at a solution. Participant says what steps they are taking and the reasons why they are taking those steps.

**Reviewing:** Self-monitoring or assessing progress as problem-solving occurs, or assessing the problem solution (e.g., checking for reasonableness) once the problem-solving process has concluded. Participant reviews work and makes corrections or assures themselves that they are doing problem correctly.

**A priori Code on Transfer**

**Transfer:** The participant mentions that what they are doing is because of what they did in the previous problem.

**Emergent Codes**

**View an example-mimic:** The participant follows what is shown in the example step-by-step and changes the numbers to theirs.

**View an example-process:** The participant looks at the example to understand the steps needed to solve the problem but does not use it as a template.

**View an example-sense-making:** The participant looks at the example problem to check if they were on the right track or to see why they were incorrect.

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## Biographical Information

Tyson Bailey completed his secondary education at Stratford High School in Nashville, Tennessee, in 1995. He then pursued higher education at Tennessee State University, also located in Nashville, where he obtained his Bachelor of Science in Mathematics, Master of Education in Curriculum and Instruction, and Education Specialist in Administration and Supervision in 2000, 2005, and 2007, respectively. He recommenced his academic pursuits, earning a Master of Science and Doctor of Philosophy in Mathematics from the University of Texas at Arlington in 2021 and 2024, respectively.